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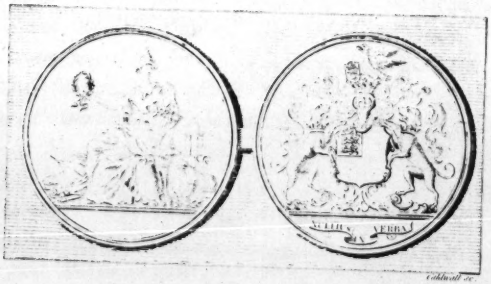

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PHILOSOPHICAL TRANSACTIONS.

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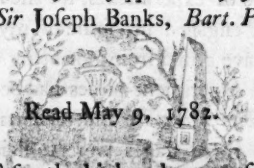

Read May 9, 1782.

A MEASURE for the higher degrees of heat, ſuch as the common thermometers afford for the lower ones, would be an important acquiſition, both to the philoſopher and the practical artiſt. The latter muſt feel the want of ſuch a meaſure

on many occaſions; particularly when he attempts to follow, or apply to uſe, the curious experiments of Mr. POTT, related in his Lithogeognoſia, and other modern writers upon ſimilar ſubjects. When we are told, for inſtance, that ſuch and ſuch materials were changed by fire into a fine white, yellow, green, or other coloured glaſs: and find, that theſe effects do not happen, unleſs a particular degree of fire has fortunately been hit upon, which degree we cannot be ſure of ſucceeding in again:—when we are diſappointed, by having the reſult at ſome times an unvitrified maſs, and at others an over-vitrified ſcoria, from a little deficiency or exceſs of heat:—when we ſee colours altered, not only in ſhade but in kind, and in many caſes deſtroyed, by a ſmall augmentation of the heat which had produced them; inſomuch, that in the gradual increaſe of the fire, a preciſe moment of time muſt be happily ſeized, in order to catch them in perfection:—and when inconveniences, ſimilar to theſe, ariſe in operations by fire upon metals and other ſubſtances:—how much is it to be wiſhed, that the authors had been able to convey to us a meaſure of the heat made uſe of in their valuable proceſſes!

In a long courſe of experiments, for the improvement of the manufacture I am engaged in, ſome of my greateſt difficulties and perplexities have ariſen from not being able to aſcertain the heat to which the experiment-pieces had been expoſed. A *red*, *bright red*, and *white heat*, are indeterminate expreſſions; and even though the three ſtages were ſufficiently diſtinct from each other, they are of too great latitude; as the brightneſs or luminouſneſs of fire increaſes, with its force, through numerous gradations, which can neither be expreſſed in words, nor diſcriminated by the eye. Having no other reſource, I have been

been obliged to content myſelf with ſuch meaſures as my own kilns and the different parts of them afforded. Thus the kiln in which our glazed ware is fired furniſhes three meaſures, the bottom being of one heat, the middle of a greater, and the top ſtill greater: the kiln in which the biſcuit ware is fired furniſhes three or four others, of higher degrees of heat; and by theſe I have marked my regiſtered experiments. But though theſe meaſures had been fully adequate to my own views, which they were not, it is plain, that they could not be communicated to others; that their uſe is confined to a particular ſtructure of furnaces, and mode of firing; and that, upon any alteration in theſe, they would become uſeleſs and unintelligible, even where now they are beſt known. And, indeed, as this part of the operation is performed by workmen of the loweſt claſs, we cannot depend upon any great accuracy even in one and the ſame furnace. It has accordingly often happened, that the pieces fired in the top of the kiln in one experiment have been made no hotter than thoſe fired in the middle in another, and *vice verſâ*.

The force of fire, in its higher as well as lower ſtages, can no otherwiſe be juſtly aſcertained than by its effects upon ſome known body. Its effect in changing colours has already been hinted at; and I have obſerved compoſitions of calces of iron with clay to aſſume, from different degrees of fire, ſuch a number of diſtinct colours and ſhades as promiſed to afford uſeful criteria of the reſpective degrees.

With this idea, I prepared a quantity of ſuch a compoſition, and formed it into circular pieces, about an inch in diameter, and a quarter of an inch thick. A number of theſe was placed in a kiln, in which the fire was gradually augmented, with as much

much uniformity and regularity as poſſible, for near ſixty hours. The pieces, taken out at equal intervals of time during this ſucceſſive increaſe of heat, and piled in their order upon one another in a glaſs tube, exhibited a regular and pretty extenſive ſeries of colours; from a fleſh-colour to a deep brownish-red, from thence to a chocolate, and ſo on to nearly black, with all the intermediate tints between theſe colours. A back being fixed to the tube, like the ſcale of a thermometer, and the numbers of the pieces marked upon it reſpectively oppoſite to them, it is obvious, that theſe numbers may be conſidered as ſo many thermometric diviſions or degrees; and that, if another piece of the ſame compoſition be fired in any other kiln or furnace, not exceeding the utmoſt heat of the firſt, it will acquire a colour correſponding to ſome of the pieces in the tube, and thus point out the degree of heat which that piece, and conſequently ſuch other matters as were in the fire along with it, have undergone.

It muſt however be confeſſed, that, for general uſe, a thermometer on this principle is liable to objection, as ideas of colours are not perfectly communicable by words; nor are all eyes, or all lights, equally adapted for diſtinguiſhing them, eſpecially the ſhades which approach near to one another; and the effects of phlogiſtic vapours, in altering the colour, may not in all caſes be eaſily guarded againſt.

In conſidering this ſubject attentively, another property of argillaceous bodies occurred to me; a property which obtains, in a greater or leſs degree, in every kind of them that has come under my examination, ſo that it may be deemed a diſtinguiſhing character of this order of earths: I mean, the *diminution of their bulk by fire*; I have the ſatisfaction to find, in a courſe

of experiments lately made with this view, that it is a more accurate and extensive measure of heat than the different shades of colour.

I have found, that this diminution begins to take place in a low red-heat; and that it proceeds regularly, as the heat increases, till the clay becomes vitrified, and consequently to the utmost degree that crucibles, or other vessels made of this material, can support. The total contraction of some good clays which I have examined, in the strongest of my own fires, is considerably more than one-fourth part in every dimension.

If, therefore, we can procure at all times a clay sufficiently apyrous or unvitrescible, and always of the same quality in regard to contraction by heat; and if we can find means of measuring this contraction with ease and minute accuracy, I flatter myself, that we shall be furnished with a measure of fire sufficient for every purpose of experiment or business.

We have, in different parts of England, immense beds of clay; each of which, at equal depths, is pretty uniform in quality throughout its whole extent. Of all the sorts I have hitherto tried, some of the purest Cornish porcelain clays seem the best adapted, both for supporting the intensity, and measuring the degrees, of fire.

For preparing and applying this material to thermometric purposes the following method is proposed.

The clay is first to be washed over, and, whilst in a dilute state, passed through a fine lawn. Let it then be made dry, and put up in boxes *,

* While the clay is thus kept dry in boxes, as well as while it continues in its natural bed, it is secure from alterations in quality, which clays in general are subject to undergo, when exposed, for a long course of years, to the joint actions of air and moisture.—In the lawns I made use of, the interstices were each less than the 100,000 part of an inch.

The

The dry clay is to be ſoftened, for uſe, with about two fifths of its weight of water; and formed into ſmall pieces, in little moulds of metal, ſix-tenths of an inch in breadth, with the ſides pretty exactly parallel, this being the dimenſion intended to be meaſured, about four-tenths of an inch deep, and one inch long. To make the clay deliver eaſily, it will be neceſſary to oil the mould, and make it warm.

Theſe pieces, when perfectly dry, are put into another iron mould or gage, conſiſting only of a bottom, with two ſides, five-tenths of an inch deep; to the dimenſions of which ſides the breadth of the pieces is to be pared down.

For meaſuring the diminution which they are to ſuffer from the action of fire, another gage is made, of two pieces of braſs, twenty-four inches long, with the ſides exactly ſtraight, divided into inches and tenths, fixed five tenths of an inch aſunder at one end, and three-tenths at the other, upon a braſs plate; ſo that one of the thermometric pieces, when pared down in the iron gage, will juſt fit to the wider end. Let us ſuppoſe this piece to have diminiſhed in the fire one-fifth of its bulk, it will then paſs on to half the length of the gage; if diminiſhed two-fifths, it will go on to the narroweſt end; and in any intermediate degree of contraction, if the piece be ſlid along till it reſts againſt the converging ſides, the degree at which it ſtops will be the meaſure of its contraction, and conſequently of the degree of heat it has undergone.

Theſe are the outlines of what appears to me neceſſary for the making and uſing of this thermometer; and it is hoped, that the whole proceſs will be found ſufficiently ſimple, and eaſy of execution. It may, nevertheleſs, be proper to take notice of a few minuter circumſtances, and to mention

ſome

some observations which occurred in the progress of the inquiry.

I. There ought to be a certainty of the clay being easily, and at all times, procurable in sufficient quantity, and on moderate terms. That this is the case with the clay here made choice of, will be evident to every one acquainted with the natural history of Cornwall, where there are beds of this clay, inexhaustible, and in too many hands to be monopolized. If this should not prove satisfactory, the author offers to this illustrious Society, and will think himself honoured by their acceptance of, a sufficient space in a bed of this clay to supply the world with thermometer-pieces for numerous ages; and he does not apprehend, that any greater inconveniences can arise to foreign artists or philosophers, from their being supplied with clay for these thermometers from this spot only, than what we now feel from being supplied with mercury for the common thermometers from the Spanish or Hungarian mines.

II. We ought to be assured also, that all the clay made use of for these thermometers is perfectly similar. For this purpose, it will be best to dig it out of the earth in considerable quantity at once, an extent of some square feet or yards in area, and to the depth of six or seven yards or more from the surface, and to mix the whole thoroughly together, previous to the further preparation already mentioned. When the first quantity is exhausted, another perpendicular column may be dug from the same bed, close to the first, to the same depth, and prepared in the same manner; by which means we may be assured of its similarity with the former parcel, and that it will diminish equally in the fire.

III. This clay, dried by the ſummer heat, or in a moderately warm room, or with more heat before a fire, has not been obſerved to differ in *degree* of dryneſs. After being ſo dried, it loſes about a hundredth part of its weight in the heat of boiling water, about as much more in that of melted lead, and from thence to a red-heat ten parts, in all $\frac{12}{100}$. Each of theſe heats ſoon expels from the clay its determinate quantity of matter, chiefly air; after which, the ſame heat, though continued for many hours, has no further effect. I had ſome hopes, that the graduation of the common thermometer might be continued, upon this principle, up to the red-heat at which the ſhrinking of the clay commences, ſo as to connect the two thermometers together by one ſeries of numbers; but the loſs of weight appears not to be ſufficiently uniform or proportional to the degree of heat to anſwer that purpoſe; for it was found to go on quicker, and bladders tied to the mouths of the veſſels in which the pieces were heated, became more rapidly diſtended, at the commencement of redneſs than at any other time. From low red-heat to a ſtrong one, ſuch as copper melts in, the loſs of weight was only about two parts in a hundred; though the difference between theſe two heats appears to be much greater than what the ſame loſs correſponds to in the lower ſtages. After this period, the decreaſe of weight intirely ceaſed.

The vapours expelled from the clay, caught ſeparately in the different degrees of heat, ſeemed, from the few trials made with them, to conſiſt of common air mixed with fixt air. They all precipitated lime-water; that which was firſt extricated, exceeding weakly; the others more and more conſiderably; but the laſt not near ſo ſtrongly as the air expelled from lime-ſtone in burning. None of them were inflammable.

IV.

IV. The thermometric pieces may be formed much more expeditiously than in the single mould, by means of an instrument used for similar purposes by the potters. It consists of a cylindrical iron vessel, with holes, in the bottom, of the form and dimensions required. The soft clay, put in the vessel, is forced by a press down through these apertures, in long rods, which may be cut while moist, or broken when dry, into pieces of convenient lengths. It was hoped, that this method would of itself have been sufficient, without the addition of the paring gage, making proper allowance, in the size of the holes, for the shrinking of the clay in drying. But it was found, that a variety of little accidents might happen to alter the shape and dimensions of the pieces, in a sensible degree, while in their soft state; so that it will be always safest to have recourse to the paring gage, for ascertaining and adjusting their breadth when perfectly dry, this being the period at which the pieces are exactly alike with regard to their future diminishing; so that if they are now reduced to the same breadth, we may be sure that they will suffer equal contractions from equal degrees of heat afterwards, whether they have been made in a mould, or by a press, or in any other way; neither is any variation in the length or thickness of these pieces of the least consequence, provided one of the dimensions, that by which they are afterwards to be measured, is made accurate to the gage.

V. It will be proper to bake the pieces, when dry, with a low red-heat, in order to give them some firmness or hardness, that they may, if necessary, be able to bear package and carriage; but more especially to prepare them for being put into an immediate heat, along with the matters they are to serve as measures to, without bursting or flying, as unburnt clay would

do. We need not be ſolicitous about the preciſe degree of heat employed in this baking, provided only that it does not exceed the loweſt degree which we ſhall want to meaſure in practice; for a piece that has ſuffered any inferior degrees of heat, anſwers as well for meaſuring higher ones as a piece which has never been expoſed to fire at all. In this part of the preparation of the pieces, it may be proper to inform the operator of a circumſtance, which, though otherwiſe immaterial, might at firſt diſconcert him: if the heat is not in all of them exactly equal, he will probably find, that while ſome have begun to ſhrink, others are rather enlarged in their bulk; for they all ſwell a little juſt on the approach of redneſs. As this is the period of the moſt rapid produce of air, the extenſion may perhaps be owing to the air having at this moment become elaſtic to ſuch a degree, as to force the particles of the clay a little aſunder before it obtains its own enlargement.

VI. Each diviſion of the ſcale, though ſo large as a tenth of an inch, anſwers to $\frac{1}{60}$th part of the breadth of the little piece of clay. We might go to much greater nicety, either by making the diviſions ſmaller, or the ſcale longer; but it is not apprehended, that any thing of this kind will be found neceſſary: and, indeed, in proceeding much further in either way, we may poſſibly meet with inconveniences ſufficient to counterbalance the apparent additional accuracy of meaſurement.

VII. The diviſions of this ſcale, like thoſe of the common thermometers, are unavoidably arbitrary; but the method here propoſed appears ſufficiently commodious and eaſy of execution, the diviſions being adjuſted by meaſures everywhere known, and at all times obtainable: for however the inches uſed in different countries may differ in length, this cannot affect

affect the accuracy of the ſcale, provided that the proportions between the wider and narrower end of the gage are exactly as five-tenths of thoſe inches to three-tenths, and the length 240 of the ſame tenths; and that the pieces in their perfectly dry ſtate, before firing, fit preciſely to the wider end. When one gage is accurately adjuſted to theſe proportional meaſures, two pieces of braſs ſhould be made, one fitting exactly into one end, and the other into the other: theſe will ſerve as ſtandards for the ready adjuſtment of other gages to the dimenſions of the original.

By this ſimple method we may be aſſured, that thermometers on this principle, though made by different perſons, and in different countries, will all be equally affected by equal degrees of heat, and all ſpeak the ſame language: the utility of this laſt circumſtance is now too well known to need being inſiſted on.

VIII. If a ſcale two feet in length ſhould be reckoned inconvenient, it may be divided into two, of one foot each, by having three pieces of braſs fixed upon the ſame plate; the firſt and ſecond, five-tenths of an inch apart at one end, and four-tenths at the other; the ſecond and third, four-tenths at one end, and three-tenths at the other; ſo that the firſt reaches to the 120th diviſion, and the ſecond from thence to the 240th.

IX. As this thermometer, like all others, can expreſs only the heat felt by itſelf, the operator muſt be careful to expoſe the pieces to an equal action of the fire with the body whoſe heat he wants to meaſure by them. In kilns, ovens, reverberatories, under a muffle, and wherever the heat is pretty ſteady and uniform, the means of doing this are too

obvious to need being mentioned. But in a naked fire, where the heat is neceſſarily more fluctuating, and unequal in different parts of the fuel, ſome precaution will be required.

The thermometer-piece may generally be put into the crucible, along with the ſubject-matter of the experiment. But where the matter is of ſuch a kind as to melt and ſtick to it, the piece may be previouſly incloſed in a little caſe made of crucible clay. The ſmallneſs of the pieces will admit of this being done without inconvenience, at leaſt in any but the ſmalleſt crucibles, as the pieces themſelves may be diminiſhed to any ſize that may be found proper, provided only that one of the dimenſions, five-tenths of an inch, be preſerved, as mentioned in *Obſ.* 4.

For the very ſmalleſt ſort of crucibles, the caſe may be put in cloſe to the crucible, ſo as to form as it were an addition to its bulk on the outſide. If it be aſked, why the caſe is not always thus put in by the ſide of the crucible? it is anſwered, that in judging of the heat of *large* crucibles from a thermometer-piece placed on the outſide of them, we may ſometimes be deceived, as the piece in its little caſe has been found to heat ſooner than the matter in the larger veſſel; but in *ſmall* ones, as the crucible and caſe are nearly alike in bulk, there is little danger of error from this cauſe.

X. Theſe thermometer-pieces poſſeſs ſome ſingular properties, which we could not have expected to find united in any ſubſtance whatever, and which peculiarly fit them for the purpoſes they are here applied to.

1. When baked by only moderate degrees of fire, though they are, like other clays, of a porous texture, and imbibe

water; yet, when saturated with the water, their bulk continues exactly the same as in a dry state.

2. By very strong fire, they are changed to a porcelain or semi-vitreous texture; nevertheless, their contraction, on further augmentations of the heat, proceeds regularly as before, up to the highest degree of fire that I have been able to produce.

3. They bear sudden alternatives of heat and cold; may be dropped at once into intense fire, and, when they have received its heat, may be plunged as suddenly into cold water, without the least injury from either.

4. Even while saturated with water in their porous state, they may be thrown immediately into a white heat, without bursting or suffering any injury.

5. Sudden cooling, which alters both the bulk and texture of most bodies, does not at all affect these, at least not in any quality subservient to their thermometric uses.

6. Nor are they affected by long *continuance* in, but solely by, the *degree* of heat they are exposed to. In three minutes or less, they are perfectly penetrated by the heat which acts upon them, so as to receive the full contraction which that degree of heat is capable of producing equally with those which had undergone its action during a gradual increase of its force for many hours. Strong degrees of heat are communicated to them with more celerity than weak ones: perhaps the heat may be more readily transmitted, in proportion as the texture becomes more compact.

These facts have been ascertained by many experiments, the particulars of which are omitted, because they would swell this paper much beyond the bulk intended.

XI. The

XI. The uſe and accuracy of this thermometer for meaſuring, *after an operation*, the degree of heat which the matter has undergone, will be apparent. The foregoing properties afford means of meaſuring it alſo, eaſily and expeditiouſly, *during the operation*, ſo that we may know when the fire is increaſed to any degree previouſly determined upon. The piece may be taken out of the fire in any period of the proceſs, and dropped immediately into water, ſo as to be fit for meaſuring by the gage in a few ſeconds of time. At the ſame inſtant, another piece may be introduced into the place of the former, to be taken out and meaſured in its turn; and thus alternately, till the deſired degree of heat is obtained. But as the cold piece will be two or three minutes in receiving the full heat, and correſponding contraction; to avoid this loſs of time, it may be proper, on ſome occaſions, to have two or more pieces, according to convenience, put in together at firſt, that they may be ſucceſſively cooled in water, and the degrees of heat examined at ſhorter intervals. It will be unneceſſary to ſay any thing further upon precautions or procedures which the very idea of a thermometer muſt ſuggeſt, and in which it is not apprehended that any difficulty can occur, which every experimenter will not readily find means to obviate.

XII. It now only remains, that the language of this new thermometer be underſtood, and that it may be known what the heats meant by its degrees really are. For this purpoſe a great number of experiments has been made, from which the following reſults are ſelected.

The ſcale commences at a red-heat, fully viſible in daylight; and the greateſt heat that I have hitherto obtained in my experiments is 160°. This degree I have produced in an air-furnace about eight inches ſquare.

Mr. ALCHORNE has been so obliging as to try the necessary experiments with the pure metals at the Tower, to ascertain at what degrees of this thermometer they go into fusion; and it appears, that Swedish copper melts at 27, silver at 28, and gold at 32.

Brass is in fusion at 21. Nevertheless, in the brass and copper founderies, the workmen carry their fires to 140° and upwards: for what purpose they so far exceed the melting heat, or whether so great an additional heat be really necessary, I have not learnt.

The welding heat of iron is from 90 to 95; and the greatest heat that could be produced in a common smith's forge 125.

Cast iron was found to melt at 130°, both in a crucible in my own furnace, and at the foundery; but could not be brought into fusion in the smith's forge, though that heat is only 5° lower. The heat by which iron is run down among the fuel for casting is 150°.

As the welding state of iron is a softening or beginning fusion of the surface, it has been generally thought that cast iron would melt with much less heat than what is necessary for producing this effect upon the forged; whereas, on the contrary, cast iron appears to require, for its fusion, a heat exceeding the welding heat 35 or 40°, which is much more than the heat of melted copper exceeds the lowest visible redness.

Thus we find, that though the heat for melting copper is by some called a white heat, it is only 27° of this thermometer. The welding heat of iron, or 90°, is likewise a white heat; even 130°, at which cast iron is in fusion, is no more than a white heat; and so on to 160° and upwards is all a white heat still. This shews abundantly how vague such a denomination must be, and how inadequate to the purpose of giving us any clear

ideas

ideas of the extent of what we have been accuſtomed to conſider as one of the three diviſions of heat in ignited bodies.

A Heſſian crucible, in the iron foundery, *viz.* about 150°, melted into a ſlag-like ſubſtance. Soft iron nails, in a Heſſian crucible in my own furnace, melted into one maſs with the bottom of the crucible, at 154°: the part of the crucible above the iron was little injured.

The *fonding* heat of the glaſs furnaces I examined, or that by which the perfect vitrification of the materials is produced, was at one of them 114° for flint-glaſs, and 124° for plate-glaſs; at another it was only 70° for the former, which ſhews the inequality of heat, perhaps unknown to the workmen themſelves, made uſe of for the ſame purpoſe. After complete vitrification, the heat is abated for ſome hours to 28 or 29°, which is called the *ſettling* heat; and this heat is ſufficient for keeping the glaſs in fuſion. The fire is afterwards increaſed, for working the glaſs, to what is called the *working* heat; and this I found, in plate-glaſs, to be 57°.

Delft ware is fired by a heat of 40 or 41°; cream-coloured, or Queen's ware, by 86°; and ſtone ware, called by the French *pots de grès*, by 102°: by this ſtrong heat, it is changed to a true porcelain texture. The thermometer-pieces begin to acquire a porcelain texture about 110°.

The above degrees of heat were aſcertained by thermometer-pieces fired along with the ware in the reſpective kilns. But this thermometer affords means of doing much more, and going further in theſe meaſures than I could at firſt even have expected; it will enable us to aſcertain the heats by which many of the porcelains and earthen wares of diſtant nations and different ages have been fired: for as burnt clay, and compoſitions in which clay is a prevailing ingredient, ſuffer no diminution

diminution of their bulk by being re-paſſed through degrees of heat which they have already undergone, but are diminiſhed by any additional heat (according to *Obſ.* V.), if a fragment of them be made to ſit into any part of the gage, and then fired along with a thermometer-piece till it begins to diminiſh, the degree at which this happens points out the heat by which it had been fired before. Of ſeveral pieces of ancient Roman and Etruſcan wares, which I have examined, none appear to have undergone a greater heat than 32°, and none leſs than 20°; for they all began to diminiſh at thoſe or the intermediate degrees.

By means of this thermometer ſome intereſting properties of natural bodies may likewiſe be diſcovered or more accurately determined, and the genus of the bodies aſcertained. Jaſper, for inſtance, is found to diminiſh in the fire, like an artificial mixture of clay and ſiliceous matter; granite, on the contrary, has its bulk enlarged by fire, whilſt flint and quartzoſe ſtones are neither enlarged nor diminiſhed. Theſe experiments were made in fires between 70 and 80° of this thermometer. A ſufficient number of facts like theſe, compared with each other, and with the properties of ſuch natural or artificial bodies as we wiſh to find out the compoſition of, may lead to various diſcoveries, of which I have already found ſome promiſing appearances; but many more experiments are wanting to enable me to ſpeak with that certainty and preciſion on theſe ſubjects which they appear to deſerve.

A piece of an Etruſcan vaſe melted completely at 33°; pieces of ſome other vaſes and Roman ware about 36°; Worceſter china vitrified at 94°; Mr. SPRIMONT's Chelſea china at 105°; the Derby at 112°; and Bow at 121°; but Briſtol china ſhewed no appearance of vitrification at 135°. The common ſort

of Chineſe porcelain does not perfectly vitrify by any fire I could produce; but began to ſoften about 120°, and at 156° became ſo ſoft as to ſink down, and apply itſelf cloſe upon a very irregular ſurface underneath. The true ſtone Nankeen, by this ſtrong heat, does not ſoften in the leaſt; nor does it even acquire a porcelain texture, the unglazed parts continuing in ſuch a ſtate as to imbibe water and ſtick to the tongue. The Dreſden porcelain is more refractory than the common Chineſe, but not equally ſo with the ſtone Nankeen. The cream-coloured or Queen's ware bears the ſame heat as the Dreſden, and the body is as little affected by this intenſe degree of fire.

Mr. POTT ſays, that to melt a mixture of chalk and clay in certain proportions, which proportions appear from his tables to be equal parts, is "among the maſter-pieces of art." This mixture melts into a perfect glaſs at 123° degrees of this thermometer.

The whole of Mr. POTT's or any other experiments may, by repeating and accompanying them with theſe thermometric pieces, have their reſpective degrees of heat aſcertained, and thereby be rendered more intelligible, and uſeful, to the reader, the experimenter, and the working artiſt.

I flatter myſelf that a field is thus opened for a new kind of thermometrical inquiries; and that we ſhall obtain clearer ideas with regard to the differences of the degrees of ſtrong fire, and their correſponding effects upon natural and artificial bodies; thoſe degrees being now rendered accurately meaſurable, and comparable with each other, equally with the lower degrees of heat which are the province of the common mercurial thermometer.

APPENDIX.

APPENDIX.

ANALYSIS OF THE CLAY OF WHICH THE THERMOMETRIC PIECES ARE FORMED.

THIS clay makes no effervescence with acids. Diluted nitrous and marine acids being boiled upon it, and afterwards saturated with fixed alkali, no precipitation or turbidness appeared. It therefore contains no calcareous earth, as that earth would have been dissolved by the acids, and precipitated from them by the alkali.

Calcined with powdered charcoal, it contracted no sulphureous smell, and the acids had no more action upon it than before. It therefore contains no gypseous matter, or combination of calcareous earth with vitriolic acid; as that acid would have formed sulphur with the inflammable principle of the charcoal, and left the calcareous earth pure, or in a state of solubility by acids.

Some of the clay was calcined with an equal weight of salt of tartar, which, for the greater certainty in regard to its purity, had been run *per deliquium*, and afterwards evaporated to dryness. The calcined mixture was boiled in water, the filtered liquor slowly evaporated, and suffered to cool at intervals. No crystallization was formed: the dry salt appeared merely alkaline as at first, and deliquiated in the air; a further proof that this clay contains no gypseous matter; for the vitriolic acid would have been absorbed by the alkali, and formed vitriolated tartar, a salt which neither liquefies in the air,

nor diſſolves eaſily in water, and which therefore would have cryſtallized long before the alkali became dry, or remained after its deliquiation.

A twentieth part of gypſum, ground with clay, was very diſtinguiſhable by both the foregoing proceſſes; producing a ſulphureous ſmell, and calcareous earth by calcination with charcoal powder; and cryſtals of vitriolated tartar by calcination with the ſame alkaline ſalt.

To ſeparate the pure argillaceous part, or that matter which in all clays forms alum with the vitriolic acid, 240 grains of this clay were thoroughly moiſtened with oil of vitriol, boiled to dryneſs, and at laſt made nearly red-hot. The mixture was then boiled in water; the earth which remained undiſſolved was treated again in the ſame manner with vitriolic acid, and this operation repeated five or ſix times. The clay was diminiſhed in the firſt operation about 70 grains; but leſs and leſs in the ſucceeding ones, and in the laſt ſcarcely two grains. The filtered liquors yielded cryſtals of true alum; but its quantity was not examined, as the produce of alum from aluminous earth is already ſufficiently known, and the quantity of aluminous earth itſelf, or its proportion to the indiſſoluble earth, was here the object. From the 240 grains of clay there remained in one experiment 98, and in another 95 grains of indiſſoluble earth; ſo that five parts of this clay conſiſt of three parts of pure argillaceous or alum earth, and two parts of an earth of a different kind.

With reſpect to the nature of this laſt earth, it is eaſier to determine negatively what it is not, than poſitively what it is; but aſcertaining the former will be a great ſtep towards the diſcovery of the latter.

That it is not calcareous, gypseous, or argillaceous, is manifest from the experiments.—It is not jasper; as this consists, in great part, of argillaceous earth, which would have been extracted by the vitriolic acid.—It is not fluor; as this, by the same acid, would have been decomposed, its own acid expelled, and a gypseous earth left.—It is not of the micaceous kind; as the peculiar aspect of these earths would readily betray them to the eye.—It is not granite; for strong fire, which granite melts in, has no effect upon this.

Nor is there any known kind of earth to which it is in any degree similar, except those of the siliceous order; and with these it perfectly agrees in all the properties, I am acquainted with, that they possess in a state of powder.

It does not vitrify or soften with pure clay, in the strongest fire I have been able to produce. Nor is it disposed to melt with the matter of Hessian crucibles; for a little of it rubbed on the inside of a crucible, and urged with strong fire, continued white, powdery, and unaltered. Thirty grains of this earth were mixed with an equal weight of dry fossil alkali, and the same quantity of a fine white quartzy sand was mixed with the same proportion of the same alkali: the two mixtures were put into two small crucibles, which were surrounded with sand in a larger one, that both might be exposed to an equal heat. They both began to melt at the same time; and at about 80° of the thermometer they had formed perfect transparent glasses.

Though these properties may not, perhaps, be thought sufficient of themselves, for determining with certainty that this substance is of the siliceous kind; yet, when joined to the negative proofs, of its not belonging to any other known order

of

of earthy bodies, they afford the fulleſt evidence which the nature of the ſubject can admit of, that the indiſſoluble part of this clay is truly ſiliceous; and conſequently that the clay conſiſts of two parts of pure ſiliceous earth, to three parts of pure argillaceous or aluminous earth.

XX. *An Analyſis of Two Mineral Subſtances*, viz. *the* Rowley-rag-ſtone *and the* Toad-ſtone. *By* William Withering, *M. D.*; *communicated by* Joſeph Prieſtley, *LL. D. F. R. S. to Sir* Joſeph Banks, *Bart. P. R. S.*

Read May 16, 1782.

TO SIR JOSEPH BANKS, BART. P. R. S.

DEAR SIR, Birmingham, Oct. 1, 1781.

I HAVE the pleaſure to lay before you an analyſis of two mineral ſubſtances by Dr. WITHERING of Birmingham, whoſe accuracy in proceſſes of this kind will, I doubt not, give you and the members of the Royal Society great ſatisfaction.

It may, perhaps, throw ſome additional light on the ſubject of theſe foſſils to inform you, that the Rowley-rag appears, by its texture before and after fuſion, and alſo by the quantity and quality of the air which it yields in fuſion, to be the ſame thing with the baſaltes with which you have favoured me from Scotland; and that the Toad-ſtone, treated in the ſame manner, appears (after the calcareous part has been diſſolved out of it) to reſemble ſome of the ſpecies of lava, except that it yields much more air. As Dr. WITHERING has ſent ſpecimens of the foſſils in their natural ſtate, I thought it might not be amiſs to preſent along with them the *glaſſy ſubſtances* into which they are reduced by fuſion.

I am, with the greateſt reſpect, &c.

J. PRIESTLEY.

TO DR. PRIESTLEY.

Birmingham,
March 13, 1782.

SIR,

I NOW ſend you the reſults of my examination of the Toad-ſtone and the Rowley-rag-ſtone; being part of a plan which I have long ſince formed for a chemical analyſis of all the ſubſtances that are known to exiſt in the earth in large quantity.

Some years ago I tranſmitted to the Royal Society an analyſis of the different marles found in Staffordſhire, which they did me the honour to inſert in their Tranſactions; if they think theſe papers likewiſe worth their adoption, I ſhall ſend them the reſults of my future inquiries.

In the courſe of experiments which this ſubject has led me to, I found it convenient to form ſome new tables, and to enlarge ſome that were leſs completely formed before. Theſe tables will be uſeful in other branches of chemical inquiry. One of them I ſubjoin to the preſent papers. The facts taken from M. MACQUER are marked with an M; thoſe with the * are the conſequence of my own experiments.

In order to ſave much repetition in future, it may not be amiſs to mention, once for all, a few particulars in the conduct of theſe proceſſes.

1ſt, By *water*, is always meant water diſtilled in glaſs veſſels, or by means of a large tin refrigeratory in Mr. IRWIN's method.

2dly, Only glaſs or china veſſels are uſed in the liquid proceſſes.

3dly, By a mortar I mean thoſe excellent ones made by Mr. WEDGEWOOD; or, as will be ſpecified at the time, a ſteel mortar

mortar tempered ſo hard that it will bear the grinding of enamel in it without diſcolouration.

4thly, Filtres are never employed, it being found impoſſible to get the quantities accurate where they are uſed. The powdery parts are allowed to ſubſide until the ſupernatant liquor becomes clear. This ſometimes requires days or weeks; but I am ignorant of a better method. By giving the veſſels a circular motion round their axes, I can greatly facilitate the ſubſiding of the ſolid contents. If the ſeparating veſſels are made like a common tart-diſh, with a ſpreading border, the liquors may be poured off very near, without diſturbing the ſediments.

5thly, Phlogiſticated alkaly means the vegetable fixed alkaly prepared by the deflagration of nitre and cryſtals of tartar diſſolved in water, and boiled with Pruſſian blue in ſuch quantity that it will not any longer precipitate an earth from an acid.

I remain, &c.

W. WITHERING.

ROWLEY-RAG.

THE ſtone which is the ſubject of the following experiments forms a range of hills in the ſouthern part of Staffordſhire. The lime-ſtone rocks at Dudley bed up againſt it, and the coal comes up to the ſurface againſt the lime-ſtone. The higheſt part of the hills is near the village of Rowley. The ſummit has a craggy, broken appearance, and the fields on each ſide to a conſiderable diſtance are ſcattered over with large fragments of the rock, many of which are ſunk in the ground. In a quarry near Dudley, where a pretty large open-

ing has been made in order to get materials for mending the roads, the rock appears to be compoſed of maſſes of irregular rhomboidal figures: ſome of theſe maſſes incloſe rounded pebbles of the ſame materials. At the diſtance of four, five, or ſix miles from the hills, as at Bilſton, Willenhall, and Wedneſbury, the Rag-ſtone is frequently found ſome feet below the ſurface in rhomboidal pieces, forming an horizontal bed of no great depth, and ſeldom of more than a few yards extent. Over the whole of this tract of country it is uſed to mend the roads, and lately has been carried to Birmingham to pave the ſtreets. Some people ſell it in powder, as a ſubſtitute for emery in cutting and poliſhing.

MORE OBVIOUS PROPERTIES.

Its appearance dark grey, with numerous minute ſhining cryſtals. When expoſed to the weather gets an ochry colour on the outſide; ſtrikes fire with ſteel; cuts glaſs; melts, though not eaſily, under the blow-pipe. Heated in an open fire becomes magnetic, and loſes about 3 in 100 of its weight.

EXPERIMENTS.

A. After three drams had been broken to ſmall pieces with a hard ſteel hammer, upon a plate of the ſame metal, it was ground to an impalpable powder in one of Mr. WEDGEWOOD's China mortars. The mortar, which had been previouſly weighed, loſt only one-third of a grain weight during this operation.

B. This powder was repeatedly waſhed with pure water, ſo as to carry off all the finer parts, and the coarſer ground again, until

until the whole was waſhed away. The waſhings were then filtered, and the powder carefully collected and dried. The water employed in the waſhings did not appear to have diſſolved any part of the ſtone; for no precipitate was formed either upon the addition of mild fixed alkaly, or of ſilver diſſolved in the nitrous acid.

C. 100 parts of this powder were put into a ſmall mattraſs, and covered with marine acid: a degree of heat was excited, and a very ſlight efferveſcence took place. Water was then added, and the mixture kept boiling for half an hour. The liquor was decanted off, and more acid added, which was boiled as before. This was decanted, and the reſiduum waſhed with water until the water came off taſteleſs. Theſe waters were added to the liquors before decanted. The powder had now an aſh-coloured appearance, and when dried weighed 80¼.

To the liquors (C) phlogiſticated fixed alkaly was added, until no more Pruſſian blue was precipitated. To effect this it took one ounce, five drams, and twelve grains of the phlogiſticated alkaly. The precipitate, when waſhed and dried, weighed 47.

E. The powder of 80¼ (C) mixed with twice its weight of foſſile fixed alkaly, was put into a black lead crucible, and expoſed to a red-heat for two hours. The heat was never ſufficient to render the maſs fluid, nor to make it adhere firmly to the crucible. The ſaline part was then waſhed away by repeated effuſions of hot water. To the remaining powder marine acid was added repeatedly, and boiled as before. The powder was now perfectly edulcorated by hot water, and when dry weighed 47¼.

The above liquors were all added to the liquor (C), and phlogiſticated fixed alkaly was dropped in, until no more Pruſſian

blue was precipitated. To effect this, half an ounce of the alkaly was required. This precipitate weighed 19; so that the whole of the Pruſſian blue weighed 66. After calcination in a crucible it was reduced to 31½, and was then wholly attracted by a magnet.

F. Mild fixed alkaly was now gradually added to the liquors after the ſeparation of the Pruſſian blue, and a white powder was precipitated. This powder, when well waſhed and dried, weighed 46½. After being expoſed to a low red-heat for ten minutes, it weighed only 32½.

G. The edulcorated powder (E) was now perfectly white; was not acted upon either by the vitriolic, nitrous, or marine acids, but readily melted into a glaſs with foſſile fixed alkaly; during the melting an effervescence took place.

H. The white powder (F) readily diſſolved in diluted vitriolic acid, and under a ſlow evaporation formed cryſtals which had the appearance and the taſte of allum.

Theſe cryſtals were then reduced to powder, and boiled in alcohol. The alcohol was decanted off, but did not appear to have diſſolved any part of the powder; nor did it afford any precipitate upon the addition of mild fixed alkaly.

CONCLUSIONS.

From theſe experiments it appears, that the Rowley-rag-ſtone conſiſts of ſiliceous earth, clay, or earth of allum, and calx of iron. From the latter muſt be deducted 11½ for the quantity of calciform iron, found by experiment to be contained in the quantity of phlogiſticated alkaly made uſe of, and then the proportions in 100 parts of the ſtone will be theſe:

Pure ſiliceous earth -	47½
Pure clay, free from fixable air	32½
Iron in a calciform ſtate -	20
	100

From this view of the component parts of this ſtone, it is not improbable, that it might advantageouſly be uſed as a flux for calcareous iron ores. The makers of iron are acquainted with ſuch ores; but never could work them to advantage, for want of a cheap and efficacious flux.

TOAD-STONE.

FROM Derbyſhire; ſent to me by Mr. WHITEHURST, who has ſo fully and ſo accurately deſcribed the mode of its ſtratification, that it is needleſs to enlarge upon that ſubject.

MORE OBVIOUS PROPERTIES.

Of a dark browniſh grey, a granulated texture; with ſeveral cavities filled with cryſtallized ſpar. It does not ſtrike fire with ſteel. It melts to a black glaſs.

EXPERIMENTS.

A. 100 parts rubbed to an extremely fine powder in a china mortar, and boiled in marine acid; the ſolution was decanted: the undiſſolved part, after proper waſhing and drying, weighed 71.

B. The

B. The undiſſolved part was rubbed with twice its weight of mild foſſil alkaly, and then expoſed to a red heat in a black lead crucible for one hour.

C. This mixed maſs was reduced to powder, and repeatedly boiled, firſt in marine, afterwards in ſtrong vitriolic acid: the reſiduum now weighed 56, and was perfectly white.

D. The liquors of exp. A. and C. being put all together, phlogiſticated fixed alkaly was added until no further precipitation enſued. This precipitate was a Pruſſian blue, which, when waſhed and dried, weighed $56\frac{5}{10}$.

After expoſure to a red-heat in a crucible for forty minutes, it weighed only 29, and was wholly attracted by the magnet.

Now the 2 oz. 5 dr. and 32 gr. of phlogiſticated fixed alkaly uſed in this experiment contain 13 gr. of calciform iron, as aſcertained by a ſeparate trial; therefore, deducting 13 from 29, we have 16 for the quantity of calciform iron obtained from the ſtone.

E. The earthy parts were next precipitated from the liquors by the addition of mild foſſil alkaly. The precipitate, when perfectly edulcorated and dried, weighed $29\frac{8}{10}$.

F. Diſtilled vinegar was added to this powder, and ſuffered to ſtand in a cool place for four hours; the vinegar was poured off, and the reſiduum repeatedly waſhed with pure water. To theſe liquors mild fixed alkaly was added, and a white precipitate ſubſided, which, when waſhed and dried, weighed $7\frac{5}{10}$.

G. To the reſiduum (F) dilute vitriolic acid was added: a ſolution took place, which ſolution, by evaporation and cryſtallization, yielded allum.

H. The part of the reſiduum (F) undiſſolved by the vitriolic acid was boiled in nitrous acid, in marine acid, and in aqua regia, without being diminiſhed; the weight of it when dried

was

was $7\frac{5}{10}$. It could not be fuſed by the greater heat of a blow-pipe, but melted into a glaſs when mixed with calcareous earth.

I. The undiſſolved part (exp. C.) was not fuſible by itſelf; nor was it acted on by vitriolic, nitrous, or marine acid. It melted into a glaſs with foſſil alkaly.

K. The precipitate of $7\frac{5}{10}$ (exp. F.) after a ſufficient expoſure to heat was put into an ounce of water: the next morning the water had a pellicle upon its ſurface, and taſted like lime-water.

CONCLUSIONS.

Hence it appears, that 100 parts of this ſpecimen of Toad-ſtone contained

C. Siliceous earth	56	$63\frac{5}{10}$
H. More ditto	$7\frac{5}{10}$	
D. Calciform iron	16	
F. K. Calcareous earth	$7\frac{5}{10}$	
G. H. Earth of allum	$14\frac{5}{10}$	
	$101\frac{5}{10}$	

From the addition of $1\frac{5}{10}$ of weight it is probable, that the ſubſtances capable of uniting with fixable air were not in the ſpecimen uſed fully ſaturated with it, as they would be after their precipitation by the mild alkaly.

Upon repeating theſe experiments with different portions of the Toad-ſtone, the quantities of the calcareous earth were found to differ a little; but nothing further appeared to invalidate the general concluſions.

A TABLE ſhewing the Solubility or Inſolubility of certain Saline Subſtances in Alcohol.

		Subſtances.	Reſults.
Vitriolic acid	Neutral	Vitriolated tartar	Inſoluble. M.
		Glauber's ſalt	Inſoluble. M.
		Vitriolic ammoniac.	Inſoluble. M.
	Metallic	Vitriol of ſilver	Inſoluble. M.
		—— mercury	Inſoluble. M.
		—— copper	Inſoluble. M.
		—— iron	Inſoluble. M.
		—— zinc	Inſoluble. *.
	Earthy	Heavy ſpar	Inſoluble. *.
		Selenite	Inſoluble. M.
		Allum	Inſoluble. *.
		Epſom ſalt	Soluble. *.
Nitrous acid	Neutral	Nitre	Soluble. M.
		Cubic nitre	Soluble. M.
		Nitrous ammoniac.	Soluble. M.
	Metallic	Nitre of ſilver	Soluble. M.
		—— mercury	Inſoluble. M.
		—— copper	Soluble. M.
		—— lead	Soluble. *.
	Earthy	Calcareous nitre	Soluble. M.
Muriatic acid	Neutral	Digeſtive ſalt	Soluble. M.
		Common ſalt	Inſoluble. M.
		Sal ammoniac.	Soluble. M.
	Metallic	Luna cornea	Inſoluble. M.
		Corroſ. Sublimate	Soluble. M.
		Muria cupri	Soluble. M.
		—— ferri	Soluble. M.
	Earthy	Muria calcarea	Soluble. M.
		—— magneſiæ	Soluble. *.
		—— aluminoſa	Soluble. *.
Vegetable acid	Neutral	Soluble tartar	Soluble. *.
		Rochelle ſalt	Inſoluble. *
		Veget. ammoniac.	
	Metallic	Verdigraſs	Soluble. *.
		Sugar of lead	Soluble. *.
Calcareous acid	Neutral	Veg. alkaly mild	Inſoluble. *
		Foſſ. alkaly mild	Inſoluble. *
		Vol. alkaly mild	Soluble. *.
		Calcareous ſpar	Soluble. *.

XXI. *New Fundamental Experiments upon the Collision of Bodies.* By Mr. John Smeaton, *F. R. S. in a Letter to Sir* Joseph Banks, *Bart. P. R. S.*

Read April 18, 1782.

TO SIR JOSEPH BANKS, BART. P. R. S.

SIR,

THE ſubjects of the incloſed tract have been the object of my conſideration for many years paſt; and as they contain ſome matters that have not only been variouſly reaſoned about, but variouſly concluded upon; if what is contained therein ſhall appear of ſuch a nature as either to eſtabliſh truth, as it appears to me; or to prompt ſome more able perſon, in reviewing the ſubject, to ſhew what links in my chain of reaſoning thereon are defective, ſo as to eſtabliſh the whole doctrine of moving bodies upon one plain conſiſtent baſis, my end will be equally anſwered in offering them to you, to be laid before the Royal Society, in caſe you ſhall think that the importance of the ſubject ſhall merit the ſame: furthermore, I hope to be forgiven, if in ſome parts of this paper I have expreſſed myſelf with more pointedneſs than I might have done, for I declare, that it was ſolely owing to my earneſtneſs that the ſubject of mechanic motions and powers ſhould be fully and freely inveſtigated, and eſtabliſhed upon grounds that ſhall be uncontrovertible.

I have the honour to be, &c.

Gray's-Inn,
April 18, 1782.

IT is univerſally acknowledged, that the firſt ſimple principles of ſcience cannot be too critically examined, in order to their being firmly eſtabliſhed; more eſpecially thoſe which relate to the practical and operative parts of mechanics, upon which much of the active buſineſs of mankind depends. A ſentiment of this kind occaſioned my tract upon *Mechanic Power*, which was publiſhed in the Philoſophical Tranſactions, vol. LXVI. for the year 1776. What I have now to offer was intended as a ſupplement thereto, and the experiments were then, in part, tried; but the completion thereof was deferred at that time, partly from want of leiſure; partly to avoid too great a length of the paper itſelf; and partly to avoid the bringing forward too many points at once. My preſent purpoſe is to ſhew, that the true doctrine of the *colliſion of bodies* hangs as it were upon the ſame hook, as the doctrine of the gradual generation of motion from reſt, conſidered in that paper; that is, that whether bodies are put into gradual motion, and uniformly accelerated from reſt to any given velocity; or are put in motion, in an inſtantaneous manner, when bodies of any kind ſtrike one another; the motion, or ſum of the motions produced, has the ſame relation to mechanic power therein defined, which is neceſſary to produce the motion deſired. To prove this, and at the ſame time to ſhew ſome capital miſtakes in principle, which have been *aſſumed* as indiſputable truths by men of great learning, is the reaſon of my now purſuing the ſame ſubject.

I do not mean to point out the particular miſtakes which have been made by particular men, as that would lead me into too great a length: I ſhall therefore content myſelf with obſerving, that the laws of colliſion, which have been inveſtigated by mathematical philoſophers, are principally of three kinds; *viz.* thoſe relating to bodies perfectly *elaſtic*; to bodies perfectly

perfectly unelastic, and perfectly *soft*; and to bodies perfectly unelastic, and perfectly *hard*. To avoid prolixity, I shall consider in each, only the simple case of two bodies which are equal in weight or quantity of matter striking one another. Respecting those which are perfectly elastic, it is universally agreed, that when two such bodies strike one another, no motion is lost; but that in all cases, what is lost by one is acquired by the other: and hence, that if an elastic body in motion strikes another at rest, upon the stroke the former will be reduced to a state of rest, and the latter will fly off with an equal velocity.

In like manner, if a non-elastic *soft* body strikes another at rest, they neither of them remain at rest, but proceed together from the point of collision with exactly one half of the velocity that the first had before the stroke; this is also universally allowed to be true, and is fully proved by every good experiment upon the subject.

Respecting the third species of body, that is, those that are non-elastic, and yet perfectly hard; the laws of motion relating to them, as laid down by one species of philosophers, have been rejected by another; the latter alledging, that there are no such bodies to be found in nature whereon to try the experiment; but those who have laid down and assigned the doctrine that would attend the collision of bodies of this kind (if they could be found) have universally agreed, that if a non-elastic *hard* body was to strike another of the same kind at rest, that, in the same manner as is agreed concerning non-elastic soft bodies, they neither of them would remain at rest, but would in like manner proceed from the point of collision, with exactly one half of the velocity that the first had before the stroke: in short, they lay it down as a rule attending all non-elastic bodies, whether hard or soft, that the velocity after the stroke will

be the ſame in both, *viz. one half* of the velocity of the original ſtriking body.

Here is therefore the aſſumption of a principle, which in reality is proved by no experiment, nor by any fair deduction of reaſon that I know of, *viz.* that the velocity of non-elaſtic *hard* bodies after the ſtroke muſt be the ſame as that reſulting from the ſtroke of non-elaſtic *ſoft* bodies; and the queſtion now is, whether it is true or not?

Here it may be very properly aſked, what ill effects can reſult to practical men, if philoſophers ſhould reaſon wrong concerning the effects of what does not exiſt in nature, ſince the practical men can have no ſuch materials to work upon, or misjudge of? But it is anſwered, that they who infer an equality of effects between the two ſorts, may from thence be miſled themſelves, and in conſequence miſlead practical men in their reaſonings and concluſions concerning the ſort with which they have abundant concern, to wit, the non-elaſtic *ſoft bodies*, of which water is one, which they have much to do with in their daily practice.

Previous to the trying my experiment on mills I never had doubted the truth of the doctrine, that the ſame velocity reſulted from the ſtroke of both ſorts of non-elaſtic bodies; but the trial of thoſe experiments made me clearly ſee at leaſt the inconcluſiveneſs, if not the falſity of that doctrine: becauſe I found a reſult which I did not expect to have ariſen from either ſort; and for the which, when it appeared from experiment, I could ſee a ſubſtantial reaſon why it ſhould take place in one ſort, and that it was impoſſible that it could take place in the other; for if it did, the bodies could not have been perfectly *hard*, which would be contrary to the hypotheſis. Of this deduction I have given notice in my ſaid tract on mills, publiſhed

lished in the Philosophical Transactions, vol. LI. for the year 1759*.

It may also be said, that since we have no bodies perfectly elastic, or perfectly unelastic and *soft*, why should we expect and bodies perfectly unelastic and *hard*? Why may not the effects be such as should result from a supposition of their being *imperfectly elastic* joined with their being *imperfectly hard*? But here I must observe, that the supposition appears to be a contradiction in terms.

We have bodies which are so nearly perfectly elastic, that the laws may be very well deduced and confirmed by them; and the same obtains with respect to non-elastic *soft* bodies; but concerning bodies of a mixed nature, which are by far the greatest number, so far as they are wanting in elasticity, they *are soft*, and *bruise*, *yield*, or *leave a mark* in collision; and so far as they are not perfectly soft they are elastic, and observe a mixture of the law relative to each; but imperfectly elastic bodies, imperfectly hard come in reality under the *same description* as the former mixed bodies: for so far as they are imperfectly hard they are soft, and either *bruise* and *yield*, or leave a mark in the stroke; and so far as they want perfect elasticity, they are non-elastic; that is to say, they are bodies imperfectly elastic, and imperfectly soft; and in fact I have never yet seen any bodies but what come under this description. It seems, therefore, that respecting the *hardness* of bodies they differ in degrees of it, in proportion as they have a greater degree of tenacity or cohesion; that is, are further removed from perfect

* "The effect, therefore, of overshot wheels, under the same circumstance of "quantity and fall, is at a medium double to that of the undershot: and as a "consequence thereof, that *non-elastic bodies, when acting by their impulse or collision,* "*communicate only a part of their original power*; the other part *being spent in* "*changing their figure in consequence of the stroke.*" Phil. Trans. vol. LI. p. 133.

softness,

ſoftneſs, at the ſame time that their elaſtic ſprings, ſo far as they reach, are very ſtiff; and hence we may (by the way) conclude, that the ſame mechanic power that is required to change the figure in a *ſmall degree* of thoſe bodies that have the popular appellation of *hard bodies*, would change it in a *great degree* in thoſe bodies that approach towards ſoftneſs, by having a ſmall degree of tenacity or coheſion. In the former kind we may rank the harder kinds of *caſt iron*, and in the latter, *ſoft tempered clay*.

While the philoſophical world was divided by the diſpute about the *old* and *new opinion*, as it was called, concerning the powers of bodies in motion, in proportion to their different velocities: thoſe who held the old opinion contending, that it was as the velocity *ſimply*, aſked thoſe of the new, How, upon their principles, they would get rid of the concluſions ariſing from the doctrine of unelaſtic perfectly hard bodies? They replied, They found no ſuch bodies in nature, and therefore did not concern themſelves about them. On the other hand, thoſe of the new opinion aſked thoſe of the old, How they would account for the caſe of non-elaſtic ſoft bodies, where, according to them, the whole motion loſt by the ſtriking body was retained in the two after the ſtroke (the two bodies moving together with the half velocity), though the two non-elaſtic bodies had been bruiſed and changed their figure by the ſtroke; for, if no motion was loſt, the change of figure muſt be an effect without a cauſe? To obviate this, thoſe of the old opinion ſeriouſly ſet about proving, that the bodies might change their figure, without any loſs of motion in either of the ſtriking bodies.

Neither of theſe anſwers have appeared to me ſatisfactory, eſpecially ſince my mill experiments: for with reſpect to the firſt, it is no proper argument to urge the impoſſibility of find-

ing the proper material for an experiment, in anſwer to a concluſion drawn from an abſtract idea. On the other hand, if it can be ſhewn, that the figure of a body can be changed, without a *power*, then, by the ſame law, we might be able to make a *forge hammer* work upon a maſs of ſoft iron, without any other power than that neceſſary to overcome the friction, reſiſtance, and original *vis inertiæ*, of the parts of the machine to be put in motion: for, as no progreſſive motion is given the maſs of iron by the hammer (it being ſupported by the anvil), no power can be expended that way; and if none is loſt to the hammer from changing the figure of the iron, which is the only effect produced, then the whole power muſt reſide in the hammer, and it would jump back again to the place from which it fell, juſt in the ſame manner as if it fell upon a body perfectly elaſtic, upon which, if it did fall, the caſe would really happen: the power, therefore, to work the hammer would be the ſame, whether it fell upon an elaſtic or non-elaſtic body; an idea ſo very contrary to all experience, and even apprehenſion, of both the philoſopher and vulgar artiſt, that I ſhall here leave it to its own condemnation.

As nothing, however, is ſo convincing to the mind as experiments obvious to the ſenſes, I was very deſirous of contriving an experiment in point; and as I ſaw no hopes of finding matter to make a *direct* experiment, I turned my mind towards an indirect one; ſo circumſcribed, however, as to prove inconteſtably, that the reſult of the ſtroke of two non-elaſtic perfectly hard bodies could not be the ſame as would reſult from the colliſion of two ſoft ones; that is, if it can be *bona fide* proved, that *one half* of the original power is loſt in the ſtroke of ſoft bodies by the change of figure (as was very ſtrongly ſuggeſted by the mill experiments); then ſince no ſuch loſs

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can happen in the collision of bodies perfectly hard, the result and consequence of such a stroke must be *different*.

The consequence of a stroke of bodies perfectly hard, but void of elasticity, must doubtless be different from that of bodies perfectly *elastic*: for having no spring the body at rest could not be driven off with the velocity of the striking body, for that is the consequence of the action of the spring or elastic parts between them, as will be shewn in the result of the experiments; the striking body will therefore not be stopped, and as the motion it loses must be communicated to the other, from the equality of action and re-action, they will both proceed together, with an equal velocity, as in the case of non-elastic soft bodies: the question, therefore, that remains is, what that *velocity must be?*—It must be greater than that of the non-elastic soft bodies, because there is no mechanical power lost in the stroke. It must be less than that of the striking body, because, if equal, instead of a *loss* of motion by the collision, it will be doubled. If, therefore, non-elastic soft bodies lose half their motion, or mechanical power, by change of figure in collision, and yet proceed together with half the velocity, and the non-elastic hard bodies can lose *none* in any manner whatever; then, as they must move together, their velocity must be such as to preserve the equality of the mechanic power, *unimpaired*, after the stroke the same as it was before it.

For example, let the velocity of the striking body before the stroke be 20, and its mass or quantity of matter 8; then, according to the rule deduced from the experiments in the tract on *Mechanic Power* (see exp. third and fourth) that power will be expressed by $20 \times 20 = 400$, which $\times 8 = 3200$; and if half of it is lost in the stroke, in the case of non-elastic soft bodies, it will be reduced to 1600; which $\div 16$ the double quantity of matter, will give 100 for the square of their velocity; the

ſquare root of which being 10, will be the velocity of the two non-elaſtic ſoft bodies after the ſtroke, being juſt one half of the original velocity, as it is conſtantly found to be. But in the non-elaſtic hard bodies, no power being loſt in the ſtroke, the mechanic power will remain after it, as before it, $=3200$; this, in like manner, being divided by 16, the double quantity of matter, will give 200 for the ſquare of the velocity, the ſquare root of which is 14. 14, &c. for their velocity after the ſtroke, which is to 10, the velocity of the non-elaſtic ſoft bodies after the ſtroke, as the ſquare root of 2 to 1; or as the diagonal of a ſquare to its ſide.

It remains, therefore, now to be proved, that preciſely half of the mechanic power is *loſt* in the colliſion of non-elaſtic ſoft bodies; for which purpoſe my mind ſuggeſted the following reflections. In the colliſion of elaſtic bodies the effect ſeemingly inſtantaneous, is yet performed in *time*; during which time the natural ſprings reſiding in elaſtic bodies, and which conſtitute them ſuch, are bent or forced, till the motion of the ſtriking body is divided between itſelf and the body at reſt; and in this ſtate the two bodies would then proceed together, as in the caſe of non-elaſtic ſoft bodies; but as the ſprings will immediately reſtore themſelves in an equal time, and with the ſame degree of *impulſive force*, wherewith they were bent in this re-action, the motion that remained in the ſtriking body will be totally deſtroyed, and the total exertion of the two ſprings, communicated to the original reſting body, will cauſe it to fly off with the ſame velocity wherewith it was ſtruck.

Upon this idea, if we could conſtruct a couple of bodies in ſuch a way that they ſhould either act as bodies perfectly elaſtic; or, that their ſprings ſhould at pleaſure be hooked up, retained, or prevented from reſtoring themſelves, when at their extreme degree of bending; and if the bodies under theſe

circumſtances obſerved the laws of colliſion of non-elaſtic ſoft bodies, then it would be proved, that one half of the mechanical power, reſiding in the ſtriking body, would be loſt in the action of colliſion; becauſe the impulſive force or power of the ſpring in its reſtitution being cut off, or ſuſpended from acting, which is equal to the impulſive force or power to bend it (and which alone has been employed to communicate motion from one body to the other), it would make it evident, that one half of the impulſive force is loſt in the action, as the other half remains *locked up* in the ſprings. It alſo follows, as a *collateral circumſtance*, that be the impulſive power of the ſprings what it may from firſt to laſt, yet as one half of the *time* of the action is by this means cut off, in this ſenſe alſo it will follow, that one half of the mechanic power is deſtroyed; or rather, in this caſe, remains locked up in the ſprings, capable of being *re-exerted* whenever they are ſet at liberty, and of producing a freſh mechanical effect, equivalent to the motion or mechanical power of the two non-elaſtic ſoft bodies after their colliſion.

Hence we muſt infer, that the quantity of mechanical power expended in diſplacing the parts of non-elaſtic ſoft bodies in colliſion, is exactly the ſame as that expended in bending the ſprings of perfectly elaſtic bodies; but the difference in the ultimate effect is, that in the non-elaſtic ſoft bodies, the power taken to diſplace the parts will be totally loſt and deſtroyed, as it would require an equal mechanic power to be raiſed a-freſh, and exerted in a contrary direction to reſtore the parts back again to their former places; whereas, in the caſe of the elaſtic bodies, the operation of half the mechanic power is, as obſerved already, only locked up and ſuſpended, and capable of being re-exerted without a further original acceſſion.

These ideas arose from the result of the experiments tried upon the machine described in my said tract upon Mechanic Power, and were also communicated to my very worthy and ingenious friend WILLIAM RUSSELL, Esq. F. R. S. at the same time that I shewed him those experiments in 1759; but the mode of putting this matter to a full and fair mechanical trial has since occurred; and though some rough trials, sufficient to shew the effect, were made thereon, prior to the offering the paper on mechanical power to the Society in 1776, yet the machine itself I had not leisure to complete to my satisfaction till lately; which I mention to apologize for the length of time that these speculations have taken in bringing forward.

DESCRIPTION OF THE MACHINE FOR COLLISION.

Fig. 1. shews the front of the machine as it appears at rest when fitted for use.

A is the pedestal, and AB the pillar, which supports the whole, C, D are two compound bodies of about a pound weight each, but as nearly equal in weight as may be. These bodies are alike in construction, which will be more particularly explained by fig. 2. These bodies are suspended by two white fir rods of about half an inch diameter *ef* and *gh*, being about four feet long from the point of suspension to the center of the bodies; and their suspension is upon the cross-piece II, which is mortoised through, to let the rods pass with perfect freedom; and they hang upon two small plates filed to an edge on the under side, and pass through the upper part of the rods. Their centers are at *k* and *l*, and the edges being let into a little notch, on each side the mortoise, the rods are at liberty to vibrate freely upon their respective points (or rather edges) of suspension, and are determined to one plain of vibration. MN is a flat arch of white wood, which may be covered with

paper, that the marks thereupon may be the more conſpicuous.

The croſs-piece II is made to project ſo far before the pillar, that the bodies in their vibrations may paſs clear of it, without danger of ſtriking it; and alſo the arch MN is brought ſo far forward as to leave no more than a clearance, ſufficient for the rods to vibrate freely without touching it.

Fig. 2. Shews one of the compound bodies, drawn of its full ſize. AB is a block of wood, and about as much in breadth as it is repreſented in height, through a hole in which the wood rod CC paſſes, and is fixed therein.

DB repreſents a plate of lead about three-eighths of an inch thick, one on each ſide, ſcrewed on by way of giving it a competent weight. *d*B*efg* repreſents the edge of a ſpringing plate of braſs, rendered elaſtic by hard hammering; it is about five-eighths of an inch in breadth, and about one-twentieth of an inch thick. It is fixed down upon the wooden block at its end *d*B by means of a bridge plate, whoſe end is ſhewn *hi*, and is ſcrewed down on each ſide the ſpring plate by a ſcrews which being relaxed the ſpring can be taken out at pleaſure, and adjuſted to its proper ſituation. *kl* is a light thin ſlip of a plate, whoſe under edge is cut into teeth like a fine ſaw or ratchet, and is attached to the ſpring by a pin at *k*, which paſſes through it, and alſo through a ſmall ſtud rivetted into the back part of the ſpring, and upon which pin, as a center, it is freely moveable.

mn ſhews a ſmall plate or ſtud ſeen edgeways raiſed upon the bridge plate, through an hole in which ſtud the ratchet paſſes; and the lower part of the hole is cut to a tooth ſhaped properly to catch the teeth of the ratchet, and retain it together with the ſpring at any degree to which it may be ſuddenly bent; and for this intent it is kept bearing gently downward, by means of a wire-ſpring *opq*, which is in reality double, the bearing

bearing part at *o* being semi-circular; from which branching off on each side the rod *cc*, passes to *p*, and fixes at each end into the wood at *q*. However, to clear the ratchet, which is necessarily in the middle as well as the rod, the latter is perforated; and also the block is cut away, so far as to set the main spring at *e* free of all obstacles that would prevent its play from the point B. The part *fg* is shewn thicker than the rest, by being covered with thin kid leather tight sowed on, to prevent a certain jarring that otherwise takes place on the meeting of the springs in collision.

Let us now return to fig. 1. the marks upon the arch MN are put on as follows. *op* is an arch of a circle from the center *l*, and *qr* an arch of a circle from the center *k* intersecting each other at *s*. Now the middle line of the marks *t*, *v*, are at the same distance from the middle line at *s* that the centers *kl* are; so that when each body hangs in its own free position, without bearing against the other, the rod *ef* will cover the mark at *t*, and the rod *gb* will cover the mark at *v*. From the point S upon the arches S*p* and S*q* respectively, set off points at an equal and competent distance from S each way, which will give the middle of the mark *w* and *x*: and upon the arch S*p* find a middle point between the mark *v* and *w*, which let be *y*; and on the other side, in like manner, upon the arch S*q* find a middle point for the mark *z*; then set off the distance S*v* or S*t* from *y* each way, and from *z* each way; and from these points, drawing lines to the respective centers *l* and *k*, they will give the place and position of the marks *a*, *b*, and *c*, *d*; and thus is the machine prepared for use.

FOR TRIALS ON ELASTIC BODIES.

For this use take out the pins and ratchets from each respectively, and the springs being then at liberty, with a short bit

of

of ſtick (ſuppoſe the ſame ſize as the rods) turn aſide the rod *gh* with the right-hand, carrying the body D upwards till the ſtick is upon the mark *w*, as ſuppoſe at o; there hold it, and with the left ſet the body C perfectly at reſt; in which caſe the rod *ef* will be over the mark *t*; then ſuddenly withdraw the ſtick, in the direction that the rod *gh* is to follow it, and the ſpring of the body D, impinging upon that of the body C, they will be both bent, and alſo reſtored; and the body C will fly off, and mount till its rod *ef* covers the mark *x*; the rod of the ſtriking body D remaining at reſt upon its proper mark of reſt *v*, till the body C returns, when the body D will fly off in he ſame manner; the two bodies thus rebounding a number of times, loſing a part of their vibration each time; but ſo nearly is the theory of elaſtic bodies fulfilled hereby, that the ſingle advantage of originally puſhing the rod *gh* beyond the mark *w*, by the thickneſs of the ſtick, or its own thickneſs, is ſufficient to carry the rod of the quieſcent body C completely to its mark *x*.

There are ſeveral other experiments that may be made with this apparatus, in confirmation of the doctrine of the colliſion of elaſtic bodies; which being univerſally agreed upon, and well known, it is needleſs further to dwell upon here; but reſpecting the application to non-elaſtic ſoft bodies, it is far more difficult to come at a fitneſs of materials for this kind of experiments, than it is for thoſe ſuppoſing perfectly elaſticity. The concluſions, however, may be attained with equal certainty.

FOR TRIALS ON NON-ELASTIC SOFT BODIES.

For this purpoſe the ratchets muſt be applied and put in order as before deſcribed, and the ſprings being both put to their point of reſt, let the body D be put to its mark *w* in the ſame

ſame manner as before deſcribed, and the body C to reſt. The body D being let go, and ſtriking the body C at reſt, in conſequence of the ſtroke, the ſprings being hooked up by the ratchets, they both move from their reſting marks *t*, *v*, reſpectively toward M: Now if they both moved together, and the rod *ef* covered the mark *c*, and the rod *gh* covered the mark *d* at their utmoſt limit, then they would truly obey the laws of non-elaſtic ſoft bodies; becauſe their medium aſcent would be to the mark *x*, which is juſt half the angle of aſcent to the mark *x*; but as in this piece of machinery, though the main or principle ſprings are hooked up, yet every part of them, and all the materials of which they are compoſed, and to which they are attached, have a degree, or more properly ſpeaking, a certain compaſs of *elaſticity*, which, as ſuch, is perfect, and no motion loſt thereby.

We muſt not, therefore, expect the two compound bodies after the ſtroke to ſtick together without ſeparating, as would be the caſe with bodies truly non-elaſtic and ſoft; but that from the elaſticity they are poſſeſſed of, they will by rebounding be ſeparated; but that elaſticity being perfect, can occaſion no loſs of motion to the ſum of the two bodies; ſo that if the body C aſcends as much above its mark *c* as the body D falls ſhort of its mark *d*, then it will follow, that their medium aſcent will ſtill be to the mark *x*, as it ought to have been, had they been truly non-elaſtic ſoft bodies; and this, in reality, is truly the caſe in the experiment, as nearly as it can be diſcerned.

After a few vibrations, by the rubbing of the ſprings againſt one another, they are ſoon brought to reſt; and here they would *always reſt* had they been truly and properly perfect non-elaſtic ſoft bodies; but here, as in the caſe of theſe bodies, by a change of the figure and ſituation of the component parts, there is expended one half of the mechanical power

of

of the firſt mover, yet in this caſe the other half is not *loſt*, but *ſuſpended*, ready to be re-exerted whenever it is ſet at liberty; and that it is really and *bona fide one half* and neither more or leſs, appears from this uncontroverted ſimple principle, that the power of reſtitution of a perfect ſpring is exactly equal to the power that bends it. And this may, in a certain degree, be ſhewn to be fact by experiment, if there were any need of ſuch a proof; for if, when the bodies are at reſt after the laſt experiment, the two rods are laſhed together near the bottom with a bit of thread, and then the ratchets unpinned and removed; on cutting the thread with a pair of ſciſſars they will each of them rebound, C towards M, and D towards N; and if they rebounded reſpectively to *z* and *y*, the mechanical power exerted would be the ſame as it was after the ſtroke, when the mean of their two aſcents was up to the mark *z*; but here it is not to be expected, becauſe not only the motion loſt by the friction of the ratchets is to be deducted, becauſe it had the effect of real non-elaſticity; but alſo the elaſticity that ſeparated them in the ſtroke, which was loſt in the vibrations that ſucceeded; neither of which hindered the mean aſcent to be to *z*; but yet, under all theſe diſadvantages in the machine (if not unreaſonably ill made) the rod *ef* will aſcend to *d*, and *gb* to *a*: and hence I infer, as a poſitive truth, that in the colliſion of non elaſtic ſoft bodies, *one half of the mechanic power reſiding in the ſtriking body is loſt in the ſtroke.*

Reſpecting bodies unelaſtic and perfectly hard, we muſt infer, that ſince we are unavoidably led to a concluſion concerning them, which contradicts what is eſteemed a truth capable of the ſtricteſt demonſtration; *viz.* that the velocity of the center of gravity of no ſyſtem of bodies can be changed by any colliſion betwixt one another, ſomething muſt be aſſumed that involves a contradiction. This perfectly holds,

according

according to all the eſtabliſhed rules, both of perfectly elaſtic and perfectly non-elaſtic *ſoft* bodies; rules which muſt fail in the perfectly non-elaſtic *hard* bodies, if their velocity after the ſtroke is to the velocity of the ſtriking body as one is to the ſquare root of 2; for then the center of gravity of the two bodies will by the ſtroke acquire a velocity greater than the center of gravity the two bodies had before the ſtroke in that proportion, which is proved thus.

At the outſet of the ſtriking body, the center of gravity of the two bodies in our caſe will be exactly in the middle between the two; and when they meet it will have moved from their half diſtance to their point of contact, ſo the velocity of the center of gravity before the bodies meet will be exactly one half of the velocity of the ſtriking body; and, therefore, if the velocity of the ſtriking body is 2, the velocity of the center of gravity of both will be one. After the ſtroke, as both bodies are ſuppoſed to move in contact, the velocity of the center of gravity will be the ſame as that of the bodies; and as their velocity is proved to be the ſquare root of 2, the velocity of their center of gravity will be increaſed from 1. to the ſquare root of 2.; that is, from 1. to 1.414, &c.

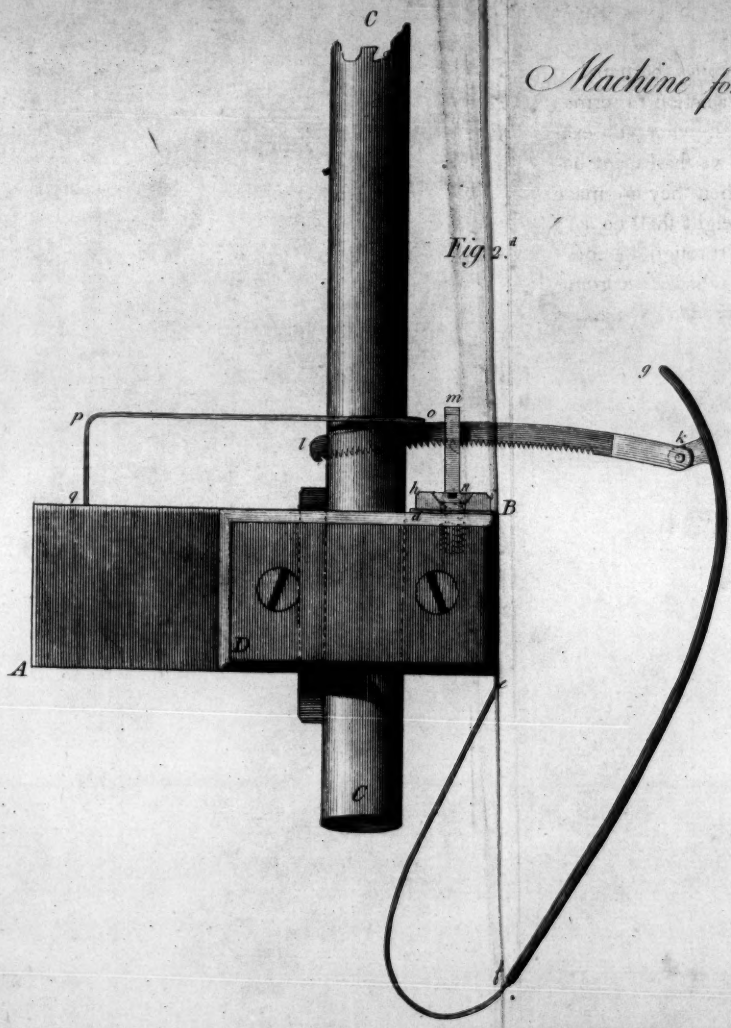
The fair inference from theſe contradictory concluſions therefore is, that an unelaſtic hard body (perfectly ſo) is a repugnant idea, and contains in itſelf a contradiction; for to make it agree with the fair concluſions that may be drawn on each ſide, from clear premiſes, we ſhall be obliged to define its properties thus: that in the ſtroke of unelaſtic hard bodies they cannot *poſſibly loſe any* mechanic power in the ſtroke; becauſe no other impreſſion is made than the communication of motion; and yet they *muſt loſe* a *quantity* of mechanic power in the ſtroke; becauſe, if they do not, their common center of gravity, as

above ſhewn, will acquire an *increaſe* of velocity by their ſtroke upon each other.

In a like manner the idea of a *perpetual motion*, perhaps, at firſt ſight, may not appear to involve a contradiction in terms; but we ſhall be obliged to confeſs that it does, when, on examining its requiſites for execution, we find we ſhall want bodies having the following properties; that when they are made to *aſcend* againſt gravitation their abſolute weight ſhall be *leſs*; and that when they *deſcend* by gravitation (through an equal ſpace) their abſolute weight ſhall be greater; which, according to all we know of nature, is a *repugnant* or *contradictory idea*.

Machine fo

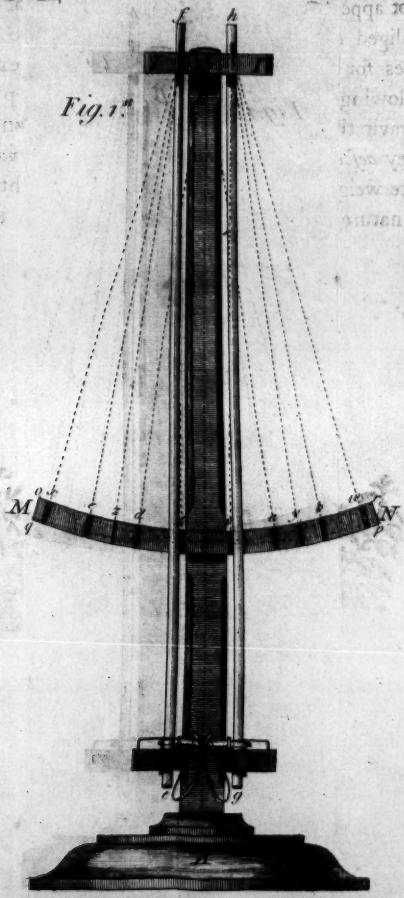
Fig. 2.d



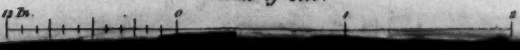
I. Smeaton D. 1784.

e for COLLISION.

Fig. 1.



Scale of Feet.



XXII. *Proceedings relative to the Accident by Lightning at* Heckingham.

LETTER FROM THE BOARD OF ORDNANCE.

SIR,

HAVING received information that, laſt ſummer, a ſtroke of lightning ſet fire to the Poor-houſe at Heckingham, near Norwich, notwithſtanding it was armed with *eight pointed conductors*, we requeſt you will communicate to us ſuch particulars relating to that fact, as may have come to your knowledge.

We are, with great reſpect,

SIR,

Your moſt obedient humble ſervants,

AMHERST.
CHARLES FREDERICK.
H. STRACHEY.
J. KENRICK.

Office of Ordnance,
2ſt December, 1781.

Sir Joſ. Banks, Bart. Preſident of the Royal Society.

Extracts from the Minutes of the Council of the Royal Society.

January 10, 1782.

THE Preſident laid before the Council a letter to him from the Board of Ordnance, acquainting him, that the Poor-houſe at Heckingham, near Norwich, had been ſtruck by lightning, notwithſtanding it was armed with eight pointed conductors; and requeſting him to communicate to them ſuch particulars relating to that fact as may have come to his knowledge.

Reſolved,

That Dr. Blagden and Mr. Nairne be requeſted to repair to Heckingham, and examine into the circumſtances of the accident, and report thereon to the Council: that they engage a draughtſman, to take ſuch drawings as may be requiſite; and that the neceſſary expences be defrayed by the Society.

February 7, 1782.

Dr. Blagden read to the Council his and Mr. Nairne's Report of the Survey made by them of the Poor-houſe at Heckingham in Norfolk, in conſequence of their appointment by a former Council. The ſaid Report was ordered to be read to the Society on Thurſday the 14th inſtant. And the Preſident was requeſted to tranſmit it immediately afterwards to the Board of Ordnance; and to deſire that they would return the drawings as ſoon as they ſhould have taken copies of them, or made ſuch other uſe of them as they might think neceſſary.

Report

Report of the Committee.

Read February 14, 1782.

To the Preſident and Council of the Royal Society.

GENTLEMEN,

PURSUANT to your reſolution, appointing us a committee to examine the Houſe of Induſtry at Heckingham in Norfolk, which had been ſtruck by lightning although it was armed with conductors, we arrived there on the 21ſt of January. Seven months had then elapſed ſince the accident, yet we had the ſatisfaction to learn, that no material changes had been made in the conductors or the building in that period; ſome laths that had been burnt, ſome bricks and pantiles which had been damaged or thrown down, were replaced; but we found means to procure diſtinct information of thoſe repairs from the workmen who had been employed to execute them. In order to communicate a clear idea of the accident, it will be neceſſary to premiſe a general account of the building; then to repreſent the manner in which the conductors were applied; and, laſtly, to deſcribe the ſtroke of lightning, with its effects.

The general form of the building is that of the Roman letter H (ſee the general plan, fig. 1.), conſiſting of a center range (Z) and two flanks (Y and X). It ſtands on a gentle riſing, which can by no means be termed a hill, with its front facing S. 9° W. To the weſtern ſide of the weſt flank, and eaſtern ſide of the eaſt flank, ſome lower buildings are annexed, ſerving as offices of different kinds; and there are two courts, one before and the other behind the houſe, together with ſome ſmall gardens and yards on each of the flanks, in all of which ſtand various detached offices, as will be eaſily conceived from the general plan (fig. 1.).

The

The body of the building, including the great house with its annexed offices, is provided with eight chimnies, the position of which is represented in the plan (fig. 1.) at the letters A, B, C, D, E, F, G, H. Of these the six first are all placed on the ridges of the roof; namely, A and B on the ridge of the west flank, C and D on the ridge of the center range, and E and F on the ridge of the east flank; but the chimney G rises from the lower part of the roof on the eastern side of the east flank; and the chimney H from the roof of an annexed office, the boiling-room, which roof is continued down from the general roof, and projects beyond it.

Both flanks (X and Y, fig. 1.) at their north and south ends are hipped off from the ridge of the roof to the eaves on each side; consequently there are eight hips, all of which are covered or *coped* with lead; the four vallies also, formed by the intersection of the center range with the two flanks (see fig. 1.) are in like manner covered with lead, which here answers the purpose of a spout. (Two of these hips are shewn in the 2d, 3d, 4th, and 11th figures at *h*, *h*, and one of the vallies at *v*, fig. 2.) Those twelve strips of lead, covering the hips and the vallies (see the general plan, fig. 1.), are all separate, not having any metalline communication with one another, as the rest of the roof consists merely of pantiles, with dropping eaves.

From the south-east corner of the east flank a wall is continued eastward (see I in the 1st and 3d figures) above 26 feet in length, having a small garden at its south side, and on the north a stable built against it as a *lean-to* (K in the 1st and 4th figures); which stable is also supported on the west by the east wall of the east flank of the building (fig. 1. at *i*). The roof of the stable being like that of a shed, slopes downward from south to north (*a* and *b* in the 4th and 11th figures); it does not reach up quite

quite to the top of the wall againſt which it reſts to the ſouthward, but is ſhorter by one courſe of bricks on edge (ſee fig. 10. and 11. at *c*); and at its junction with the wall a *flaſhing* of lead is carried along horizontally (from *c* to *d*, fig. 11.) 25¾ feet in length.

We conclude this general account of the building with the dimenſions of its principal parts.

	Ft.	In.
Length of the center range (Z fig. 1.) to the flanks - - - -	108	9
Length of each flank (X and Y fig. 1.) -	159	7
Breadth of the center range, and of each flank,	31	4
Height from the ground to the bottom of the hips ($g_2 g$, fig. 3. and 4.) - - about	20	6
Height from the ground to the top of the ridge (*f*, fig. 2. and 3. *e*, fig. 4.) - about	34	0
Height of the chimnies above the ridge of the roof (as E, fig. 4.) - - about	3	6
Length of each hip (from *f* to *g*, fig. 3. 4. and 11.) - - - about	27	0
Height of the wall (I) ſupporting the ſtable (fig. 3.)	16	0
Height of the eaves of the ſtable above the garden to the northward of it (ſee general plan, fig. 1.)	7	0
Length of the ſtable on the outſide -	26	2
Breadth of the ſtable on the outſide -	15	5

To all the eight chimnies which have been deſcribed we found iron rods affixed, reaching between four and five feet above the top of the chimney, pointed at the upper end, and tapering about ten inches to that point. Each rod or bar was nearly ſquare, meaſuring, upon a mean, about half an inch one way, and four-tenths of an inch the other, with the angles

juſt rounded off. Theſe conductors were continued down the building by a ſucceſſion of ſimilar bars of iron, in general from ſix to eight feet long, joined to one another by two hooks and nuts (ſee fig. 12); that is, the correſponding ends of each bar being formed into a hook bent at right-angles, the hook of the uppermoſt went into a hole of the lowermoſt, where it was faſtened with a nut, and the hook of the lowermoſt went into a ſimilar hole of the bar above, where it was fixed in the ſame manner; the length of each of theſe joints, from nut to nut, was about two inches.

Though there were eight of theſe conductors reaching above the chimnies, yet they had only four terminations below. For the conductors to the two chimnies D and E (fig. 1. and 2.) being continued toward each other along the roof, united in the valley over the lead gutter there (at L in the 1ſt, 2d, and 3d figures), and from that point only one conductor was continued down the valley toward the ground. In like manner the two conductors from the chimnies A and C (fig. 1.) united in the valley of the roof between them, and were carried down toward the ground as a ſingle rod. All the three conductors from the chimnies F, G, and H, ſucceſſively joined together (ſee M, N, fig. 1.), and only a ſingle rod was continued from them down the lower part of the building. Laſtly, the conductor from the chimney B (fig. 1.) went down ſingle all the way, without having formed a junction with any other.

As the conductors, therefore, in their paſſage down the building, were thus reduced to four, we are now to ſhew their four terminations. And, firſt, that from the chimney B, being the ſimpleſt, was carried down the weſtern ſide of the weſt flank, till it came very near the ground, when it entered a ſmall channel of brick-work, through which it was continued under the pavement into a narrow bricked drain, leading

through the wall of a privy (at O, fig. 1.) into which the drain diſcharges itſelf. The conductor having paſſed, in the drain, through the hole in the privy wall, was continued about three feet into the open ſpace under the ſeat of the privy, where it terminated in air, none of the ſolid work being nearer its end than ſix inches. As this drain is conſtructed to receive the foul water from the yard, and one of the water-cocks is near, ſome moiſture will moſt commonly be found in it; but the ſtones ſlope ſo rapidly at the termination of the drain in the privy, that any water which runs thither muſt be immediately carried off. This conductor, as well as all the others, was kept in its place near the wall in its paſſage down, by ring-ſtaples driven into the wall.

The ſecond termination we ſhall deſcribe is that belonging to the chimnies F, G, H (fig. 1.). The conductor from the chimney F in paſſing down the roof was joined by that from the chimney G (at M, fig. 1.) and afterwards by the rod from the chimney H (at N, fig. 1.); thence the iron was continued down till it came near the bottom of the wall, where it was turned off along the pavement toward a ſink (fig. 1. Q) not quite two feet diſtant, through the ſide of which it was carried, and projecting four inches into the open ſpace of the ſink, there terminated in air. The ſink is built of brick, one foot nine inches deep, and two feet and a half ſquare within; and into its ſouth ſide is fitted an iron grate, of the ſame length as the ſide externally, and about ſeven inches deep, the lower part of which lies on the bare ſoil. Through the middle of this grate the conductor paſſes, reſting in contact with one of the bars. From its termination to the bottom of the ſink is not leſs than twelve inches; and the bottom, which is of brick, ſlopes ſo much, that water can never lie upon it, there being a large

drain on the further ſide, which leads off from the bottom of the ſink.

The third termination to be inveſtigated is that formed by the conductors from the chimnies A and C (fig. 1.). Theſe, after joining in the valley which lies between them, ran down, as a ſingle rod, over the lead covering the valley, paſſed through a hole in that lead, where it projects over as a ſpout, and deſcended in the angle formed by the interſection of the weſt flank with the front of the center range (S, fig. 1.). Being arrived within eight inches of the ground, it entered a narrow channel of brick-work, through which it was conveyed into a ſmall cloſe drain or gutter, where it terminated, with a hooked end, in contact with one of the ſide bricks. It touched nothing ſolid, therefore, in its courſe under the ground, but brick-work. The ſmall drain in which it terminated was ſo placed as not to receive much moiſture; and this drain led into the ſide of a grated ſink (U, fig. 1.), at the bottom of which the great drain of the fore-court begins.

Of the ſeveral conductors that have been hitherto conſidered, the different parts of the building to which they were affixed, and their reſpective terminations, very accurate drawings were made on the ſpot; but as theſe conductors were more diſtant from the ſtricken end of the houſe than thoſe which remain to be deſcribed, and alſo ſhewed no marks of having been affected by the ſtorm, we thought it unneceſſary to enter into a more circumſtantial detail regarding them; eſpecially as, if any further particulars ſhould appear hereafter to be of conſequence, it will at all times be eaſy to refer to the original drawings and notes.

We proceed now to examine the fourth termination, by which the conductors from the chimnies D and E (ſee fig. 1.

and

and 2.), being thoſe neareſt the ſtricken corner, were carried under the ground. The conductor of the chimney D, from its upper point to its final termination, conſiſted of ten bars, into the ſixth of which the conductor coming from the chimney E was faſtened by its fourth bar, reckoning from the top (at L, fig. 1. and 2.). This junction was made by a hook at the lower end of that fourth bar of the conductor from the chimney E, which hook was received into a hole of the above-mentioned ſixth bar, and fixed there by a nut underneath. Here was, therefore, only one hook and nut, inſtead of two as in the common joints. Alſo at the top of this ſixth bar of the conductor from the chimney E, where it united with the fifth bar, only one hook and nut were employed to form the junction, the other hook appearing never to have been put into its correſponding hole. In this ſame ſixth bar, above the hole into which the conductor from the chimney E was inſerted, we found four other ſpare holes, which were left quite empty.

Tracing the conductor downward from this point of union, we found it deſcend over the lead of the valley, to the ſurface of which it gradually approached, till at a hole made on purpoſe (*m*, fig. 2.) it paſſed through the lead, whence it was continued down the angle formed by the interſection of the eaſt flank with the front of the center range (T, fig. 1. 2. and 5.). It no where touched the wall of the building, but was kept in its place by ring-ſtaples (*p*, *p*, fig. 3.). Being arrived within two or three inches of the ground, it entered into a channel of brick, encloſed on all ſides (at *e*, fig. 5.), in which it was continued down to the arch of the great drain of the fore-court (*x*, fig. 5.); here, having paſſed through a hole in the haunch of the arch (*y*, fig. 5. and 7.), it was bent off from the houſe through the middle of the drain, and

ultimately terminated in contact with the bricks at the bottom of it (at *z*, fig. 5. and 7.). This conductor, therefore, in its paſſage downward, did not communicate, till it reached the bottom of the drain, with any thing better able to carry off electricity than maſonry or timber; for the iron-ſtaples faſtening it to the wall, and the lead lining the valley, were themſelves in contact with ſuch ſubſtances only.

As this drain, then, is the real termination of the conductor, it muſt now be more attentively conſidered. It begins at the weſtern ſink of the fore-court U (fig. 1.); thence it is continued (V, V, fig. 1.) with a proper declivity to the eaſtern ſink W (fig. 1. 5. and 6.); it then runs under the eaſt flank of the houſe (V2, fig. 1. and 5.), and ends beyond it in the ſide of the ceſs-pool P (fig. 1.). From the grating on the ſink U to that on the ſink W (fig. 1.) is 89 feet, and thence to the ceſs-pool P near 69 feet; the breadth of the drain at bottom (*z*, fig. 7.) is 14 inches; its height to the ſpring of the arch (fig. 7.) 16 inches, and to the crown of the arch (*x*, fig. 5. and 7.) 23 inches. When we ſaw it, the moiſt filth, or ſludge, at bottom (*z*, fig. 7.) was two or three inches deep; but when the court is overflowed, as the two grates (at U and W, fig. 1.) are laid on purpoſe to receive the ſuperfluous moiſture, there muſt be ſome run of water through it. We eſtimated the fall of the drain, from the eaſtern ſink W (fig. 1.) to its termination in the ceſs-pool P, at two feet. The ceſs-pool itſelf reſembles a well, walled round in the inſide, and has foul water ſtagnating at the bottom, which cannot riſe above a certain height on account of a large drain, leading from it into the great reſervoir (at R, fig. 1.), out of which the foul water is ultimately pumped. When we examined this ceſs-pool, the water in it ſtood even with the bottom of that great drain, conſequently was almoſt

as

as high as it could be, unleſs the drains ſhould at any time be flooded; and upon meaſuring the diſtance from the bottom of the drain coming from the fore-court (V2, fig. 1.) where it terminated in the ſide of the ceſs-pool, down to the ſurface of the water ſtagnating in the ceſs-pool, we found it 3½ feet. This interval, therefore, of three feet and a half muſt be paſſed through, to form a communication between the water in the drain, and that in the ceſs-pool. The drain is firmly built of brick and mortar (ſee the ſection of it, fig. 7.). To determine the nature of the ſoil in which it is laid, a hole was dug in the fore-court ſeven feet deep, where we found nothing but ſand, at this time pretty moiſt, with a few pebbles. There is reaſon to believe, however, from the ſoil of an adjacent declivity to the northward, that below the ſand, perhaps at the depth of 15 or 16 feet, a bed of clay would be found.

Againſt the eaſt flank, near the corner T (fig. 1.), there riſes a leaden pipe with a cock (O, fig. 2.), to which the water is conveyed from a raiſed ciſtern (ſee *r*, fig. 1.) in one of the detached offices of the back-court. A main of lead from the ciſtern, which is itſelf of that metal, after ſending out pipes to ſome other cocks, and paſſing through the cellars of the houſe, comes into the fore-court about four yards from the corner T (fig. 1.) and is carried over the drain at the diſtance of about a foot above its crown, and eight inches below the ſurface of the ground. Here it divides into two branches, one of which goes ſtraight to the cock at O (fig. 2.), and the other runs weſtward, to ſupply a ſimilar cock in the oppoſite corner. We meaſured the diſtance of theſe pipes and cocks from the conductors, and found that they came no where nearer than five feet and an half.

Such

Such were the conductors that, in the month of June, 1777, ſeveral years after the Houſe of Induſtry had been built, were erected with the hope of guarding it from lightning. The iron of which they were formed had in that time acquired a coat of ruſt, ſuch as might be expected from four years expoſure to the air. On the 17th of June, 1781, after a ſhowery forenoon, a heavy cloud having riſen from the S. W. brought on a ſevere thunder-ſtorm, attended with ſuch heavy hail and rain, that the court before the houſe was overflowed. At length, about three in the afternoon, when this ſtorm had already laſted 15 or 20 minutes, a ſingle and very loud exploſion was heard, like the report of a cannon, which exceedingly terrified all the people in the houſe, and affected three of the paupers ſo much that they fainted. At the ſame time a great light was perceived, which ſeemed, as they expreſſed it, to come in at the windows, and ſtill more at the doors of the rooms, like a ſheet of fire. Within one or two minutes, the ſouth-eaſt corner of the eaſt flank of the building was obſerved to be on fire, the flame burſting out at the bottom of the hip (ſee *g* in fig. 3. 4. and 11.). By the briſk exertions of the people in the houſe, this fire was quickly extinguiſhed; and the court was ſo overflowed, that they procured ſufficient water for that purpoſe by means of a hole which they dug near the burning corner of the building. The ſtorm, and eſpecially the rain, continued ſome time after the ſtroke, but not with ſuch violence as before. At the moment of the exploſion it was nearly calm; but the wind had been ſouth-weſterly all day, and the ſky was obſerved to be clearing in that quarter about the time of the accident.

To come at the fire, in order to extinguiſh it, the lead had been rolled off the bottom of the hip, and ſome bricks thrown down, all of which were replaced when we arrived at Hecking-

ham; but as the men, who had gone up to the corner of the houſe on the firſt appearance of the fire, ſeemed to recollect very well the ſtate in which they had found the lead and tiles at that moment, they were deſired to put every thing in the ſame ſtate to the beſt of their memory. With this view they turned back the lead at the bottom of the hip on its ſouth ſide, ſo that the ſouth-weſt face of the hip-pole might be ſeen, and threw down a few tiles, after ſetting one on edge againſt the hip-rafter. The lightning then, if ſuch evidence be admitted, had raiſed up that corner of the lead to the breadth of about ſix inches at the bottom, and diſplaced ſome tiles. An effect of this kind upon the lead, is one of the commoneſt facts obſerved in buildings that have been ſtruck by lightning. It ſo happened, that the piece of lead which we found on the bottom of the hip at Heckingham, had upon it ſeveral impreſſions or pits; concerning which various opinions were entertained, till an experiment, made ſince our return to town, ſeems to have put it beyond doubt, that they are nothing but marks of large ſhot, ſuch as might have been produced by firing, with a large fowling-piece, at a bird ſitting on the corner of the houſe. All the people who aſſiſted in extinguiſhing the fire agreed, that on the eaſtern ſide of the hip, the lead remained, after the ſtroke, in its uſual ſituation.

On removing entirely the lower part of the lead, no kind of damage was ſeen on the wood of the hip-pole, except that near the lower end it was ſlightly ſcorched in one place, apparently by the flame which had burſt forth from below; the ſpike-nail which had faſtened the lead to it appeared perfectly ſound, and even the hole made by that nail in the wood was neither burnt nor ſplintered. This hip-pole was ſupported, at its proper diſtance from the hip-rafter, by an iron-ſtrap, or holdfaſt, which was driven

driven into the timber making the tie of the angle, through the bevelled end of the hip-rafter, juſt without the part where the tenon of the latter is received into the mortiſe of the former (*a*, fig. 8.). Here it was that the fire ſeemed to have begun, though neither the holdfaſt itſelf, nor the hip-pole reſting upon it, ſhewed any ſigns of the lightning. From the place into which this holdfaſt was driven (*a*, fig. 8.) to the outer end of the angle-tie (*b*, fig. 8.) there was a conſiderable loſs of ſubſtance, occaſioning a large hole; but the ſides of the hole within were ſo ſmooth, and ſo little charred, ſhewing plainly the grain of the wood, that it was ſcarcely poſſible to ſuppoſe the whole had been *burnt* out; we conjectured, therefore, that a large ſplinter had been forced off by the lightning at this place, and, in the ſame moment, the tenon of the hip-rafter ſet on fire where it enters the mortiſe. Indeed, unleſs ſome opening had been made by forcing out ſuch a piece, it does not appear how the fire could have burnt, for want of air, in a part that is always ſo cloſely joined by builders: and yet, in this confined place, the tenon of the hip-rafter was ſo far conſumed, that a ruler could be thruſt in, almoſt to the further extremity of the mortiſe. From this ſpot the flame ſeems to have iſſued out eaſtward, between the tie of the angle and the wall-plate (*c*, *c*, fig. 8.) ſcorching all the timbers it could reach, and ſetting fire to the laths; but the miſchief it had done was very trifling (ſee fig. 8.).

Juſt beneath the abovementioned hole at the end of the angle-tie (*b*, fig. 8), is the extremity of the wall-plate which lies upon the eaſtern wall of the eaſt flank (*d*, fig. 8.). The end of this wall-plate was rent in a remarkable manner (*e*, *e*, *e*, *e*, fig. 8.), and ſeveral of the fiſſures were continued ſome way upon the ſides (*d* and *f*, fig. 8.). Though the other timbers we have

mentioned are of fir, the wall-plate is of ſolid oak; and the violence done to its extremity was ſuch, that we could not doubt but it had been occaſioned by the lightning.

Under this end of the wall-plate there was a crack in the ſouth face of the corner (*m*, fig. 8.), which went down four courſes of bricks, and then terminated abruptly (*m*, fig. 3.). The three external courſes of bricks above this crack were new, and projected out much farther than the others, to form the cornice of the wall. Whether the bricks of the old cornice had been damaged or thrown down by the ſtroke, we could not learn with certainty; but the general report among the people we conſulted was, that they had not, and were only taken down to extinguiſh the fire: this opinion ſeemed probable from the want of marks on the lip-pole which projected out with the cornice, and the appearance of ſuch ſtrong effects of lightning on the wall-plate which lay within any part of the projection; whence it might be concluded, that the lightning paſſed within the cornice, and no where through it. Between the bottom of the wall-plate, and the top of the crack where it appeared to begin at the foot of the cornice (*m*, fig. 3. and 8.), lay two inner courſes of bricks (*o* and *p*, fig. 8.) covered by the cornice. Some damage had evidently been done to the bricks in this part, though we could not diſtinctly trace the progreſs of the lightning through them.

Beneath the eaſt edge of the wall-plate, and ſeparated from it in like manner by two courſes of bricks, a ſimilar crack deſcended from the bottom of the cornice (*l*, fig. 4. and 8.) on the eaſt face of the corner, and went through ten courſes of bricks till it reached the top of the wall that ſupported the ſtable. Here the three bricks next the houſe, it is ſaid, were ſhivered into pieces as ſmall as nuts, but not thrown off (*c*, fig. 10. and 11.). The cracks in the bricks on both faces of the

corner remained, having only been filled up with mortar; but new bricks were put in the place of the three that had been broken on the wall. All the workmen we saw agreed in opinion, that no iron cramps, or other metal, had been used in the brick-work.

Beginning from these three shivered bricks on the top of the wall, three courses of pantiles on the roof of the stable, in the direction downward, were in great measure broken or displaced, except about two feet of the lower end of the courses, near the eaves, where the tiles remained untouched (see *c*, *m*, *g*, *q*, fig. 10. and *c*, *q*, fig. 11.). All these pantiles rested upon laths, which were fastened to the rafters of the roof by iron nails about eleven inches asunder. Within the stable, and almost underneath the spot where the damage to the pantiles ceased, a saddle hung, at the time of the accident, by a nail driven into the wall at that west end of the stable, which was also the eastern wall of the east flank of the house (*n*, fig. 10.). As this saddle, being much torn by the lightning, seems to have been the step by which it passed through the stable, the respective situations of all their parts shall be minutely described.

The stable in its inside is 25 feet long (from *r* to *s*, fig. 9.), 13 feet broad (from *t* to *u*, fig. 9.), 15½ feet high on its south side (from *w* to *x*, fig. 10.), and 7½ feet on the north (from *y* to *z*, fig. 10.). At the west end is a stall for one horse (*s*, fig. 9.). Near the middle of the north wall is a drain (*y*, fig. 9. and 10.), which terminates just without the wall in the garden (*k*, fig. 9. and 10.). Against the west end of the stable, a shelf (*e*, *f*, fig. 10.) was supported by two nails underneath (*f*, fig. 10.). Seven inches and a half below this shelf was a nail, on which the saddle hung by one of its stirrups (*n*, fig. 10.). The breadth of the shelf was near one foot and an half; its

length (from *e* to *f*, fig. 10.) two feet five inches; and the pantiles ſeem to have been diſplaced a little farther down on the roof (*g*, fig. 10.) than the line correſponding perpendicularly with the north end of the ſhelf (*b*, fig. 10.). Neither the ſhelf, nor the nails ſupporting it, which were both near its north end, ſhewed any ſigns of injury; whence it may be conjectured, that if the lightning took its courſe this way, it paſſed obliquely between the ſaddle and the roof, ſo as to miſs the edge of the ſhelf, leaving it to the ſouthward. The upper part of the ſouth ſide of the ſtable was boarded off from the reſt, to form a hay-chamber, which occupied ſo large a portion of the roof (from *x* to *m*, fig. 10.), that the boards of the perpendicular partition (at *o*, fig. 10.) came within ten inches of the nail on which the ſaddle hung. Theſe boards were faſtened to the uprights of the partition, all the way down from the roof, by nails about ſix inches aſunder, conſequently ſome of thoſe nails muſt have been within ten inches of the ſtirrup-iron as it hung on the nail in the wall (*o* and *n*, fig. 10.). No tokens of the lightning could be diſcovered on thoſe boards, or the nails faſtening them; we could not, therefore, be certain, whether any part of it had paſſed that way. The nail which ſupported the ſaddle was equally free from marks; but one of the ſtirrup-leathers was much torn and burnt, and a large piece of the leather was ſtripped off the ſeat of the ſaddle, beſides other damage done to it in that part. One of the ſtirrup-irons, likewiſe, exhibits ſome appearances of fuſion on the arch through which the ſtirrup-leather paſſes. This iron, as well as the ſtirrup-leather, being the only damaged parts of the ſaddle that remained, we have brought for your inſpection.

It muſt be evident that we derived the knowledge of moſt of theſe circumſtances relative to the effects of the lightning upon and within the ſtable from information, the damages having

been repaired before our arrival. As the workmen present, however, agreed pretty well in their testimony, and it was corroborated by every thing that appeared, we desired them to replace all the parts as they were left by the accident, and thence made the descriptions and drawings. We gave directions that a man, accustomed to the stable, should hang up the saddle there in the usual manner, and then ascertained the following measures:

	Ft.	In.
From the nails supporting the shelf in the stable (at *f*, fig. 10.) to the nearest nails of the pantile laths of the roof (about *b*, fig. 10.) - -	1	3
From the south end of the shelf (*e*, fig. 10.) to the roof over it (*m*, fig. 10.) - -	2	1
N. B. The south end of the shelf was fixed to the partition-boards of the hay-chamber (*e*, fig. 10), and the two nails under its north side (*f*, fig. 10.) were 5¼ inches apart.		
From the nearest of the nails supporting the north end of the shelf (*f*, fig. 10.) to the nail on which the saddle hung (*n*, fig. 10.) - -	0	8
Length of the stirrup-iron below the nail (*n*, fig. 10.) - - - -	0	3½
Length of the stirrup-leather, from the stirrup-iron to the seat of the saddle - -	1	9
Breadth of the seat of the saddle - about	0	8
Distance from the lower side of the seat, as the saddle hung, to the bottom of the lowest stirrup-iron (*p*, fig. 10.) - - - -	2	0
Distance from the lowest stirrup-iron (*p*, fig. 10.) to the floor of the stable (near *d*, fig. 10. and 9.) - - - - about	3	6

As

As the ſaddle was thus placed by recollection, the girths reached from it to the ground (*d*, fig. 10. and 9.); but neither theſe girths, nor any other part of the ſaddle, except one ſtirrup-iron, one ſtirrup-leather, and the ſeat, were ſaid to have been damaged by the accident.

From the quantity of rain which fell in the thunder-ſtorm, the ſtable was overflowed with water, which gradually ſunk into the drain (at *y*, fig. 9. and 10.). The leather ſtripped off the ſeat of the ſaddle was found in the ſtable near this drain; whether thrown there originally, or carried by the water, is uncertain. From the point of the floor immediately under the ſaddle, to the neareſt part of the drain, was about 12½ feet; the width of the drain (*y*, fig. 9.) 14 inches; its length through the wall to the edge of the hole or ſink into which it diſcharges itſelf 18 inches, and the depth of the ſink from the bottom of the drain about one foot and an half. As this ſink was merely a hole, without any drain leading *from* it, and was bricked at the ſides, the water could not paſs off by the drain of the ſtable any faſter than it could ſoak through the looſe ſoil at the bottom of the ſink. And it is evident, from this conſtruction, that the earth under the ſink will uſually be ſome of the wetteſt near the building, and be impregnated with ſalts from the ſtale of the horſes.

Except the marks which have been already deſcribed, we could not find on any part of the ſtable, either within or without, the leaſt veſtige of the lightning. We particularly examined the lead *flaſhing* on the top of the roof (from *c* to *d*, fig. 11.), and the hay-chamber immediately under the three broken bricks and the diſplaced pantiles, but in vain. There was a hook fixed in the wall, 15 inches below the nail on which the ſaddle hung, and ſo exactly underneath, that the ſtirrup-leather may be

be ſuppoſed to have covered it; but this alſo appeared to be perfectly untouched. After making every poſſible inquiry, we could not determine by evidence, whether the ſtirrup-leather which is ſo ſinged and torn was the upper or the lower one at the time of the accident. Much leſs could we get information of the reſpective poſitions of the two ſtirrup-irons. But, whatever their ſituation may have been, as ſo few ſteps were to be traced, it would ſeem that the lightning muſt have jumped over at leaſt one long interval in its paſſage through the ſtable.

About ſeven feet from the ſtricken corner of the building, and almoſt two feet from the neareſt part of the roof of the ſtable, is a window (A, fig. 4. 10. and 11.) being the ſouthernmoſt of the upper range on the eaſt face of the flank. It has thirty ſmall panes of glaſs, ſet in lead. We were informed, that about half of theſe had been broken by the accident, chiefly on the ſide next the corner; and that the fiſſures ran in general horizontally, nearly parallel. Very little, if any, of the glaſs was forced out. As we could not diſcover any trace of the lightning directly toward this window, a ſuſpicion aroſe, whether it might not have been broken rather by the general concuſſion than by any immediate ſtroke.

Having examined all the marks that appeared between the bottom of the ſtricken hip and the ground, our next inquiries were directed to the top of the hip (*f*, fig. 2. 3. 4. and 11.). Here the upper plate of lead (*e*, fig. 4.) which ſerved as a capping to the junction of the hip with the ridge of the roof, being taken off, we found, on its under ſurface, three diſtinct marks of fuſion; and on the upper ſurface of the ſheet of lead which it covered three correſponding marks, ſo exactly ſimilar, that the two ſurfaces of lead ſeem to have touched one another in a melted ſtate. Theſe fuſed ſpots are juſt in the bend of the lead,

lead, anſwering to the obtuſe angle formed between the hip and the roof (*f*, fig. 4.). We obtained leave to bring away both the pieces of lead, and now preſent them for your inſpection. The workmen who examined the timber underneath reported, that it was not damaged; nor were any other ſigns of the lightning perceived in the whole length of this ſtrip of lead from the top to the bottom of the hip. In the pieces of lead which exhibit the melted ſpots on one ſurface, the other ſurface is perfectly clear of all marks, though the latter was, in the uppermoſt plate of lead, that which had lain expoſed to the clouds. Neither of them is melted to any depth into the ſubſtance of the metal.

As both extremities of the hip, therefore, were, in ſome degree at leaſt, affected by the lightning, we proceeded to aſcertain their diſtances from the neareſt conductor, which was that affixed to the chimney E (fig. 2. and 4.). Having determined the neceſſary meaſures, and calculated the hypotenuſe, the diſtance from the point of the conductor to the beginning of the lead on the top of the hip (*e*, fig. 4.) came out 42 feet and a quarter; thence to the bend where we found the marks of fuſion (from *e* to *f*, fig. 4.) was five or ſix inches more; and as the hip meaſured about 27 feet in length, the diſtance from the conductor to the bottom of the hip (*g*, fig. 4.) may be called 69 feet. From the top or bottom of the hip, to the neareſt part of the conductor as it ran downward, the diſtances were not a foot leſs than theſe meaſures. We then took down the uppermoſt rod of the conductor, and carefully examined it, eſpecially at the point, and at the hook and ſcrews by which it had been joined to the ſecond rod; but could no where diſcover the leaſt mark of fuſion or other injury. At the bottom of this conductor, however, where, having joined that from the chimney

D,

D. it terminated in the drain (ſee the general plan, fig. 1. and z, fig. 5.), a ſmall bright ſpot appeared on one of the angles. As ſome ſuſpicions were entertained, whether this mark might not have been occaſioned by the lightning, we cut off the end of the rod, and have brought it hither for public examination.

Where this conductor entered its channel at the corner of the court (ſee T, fig. 1. and *e*, fig. 2. and 5.), the ground is raiſed ſo much above the grate of the ſink (W, fig. 1. and 5.) that, though the court was overflowed, it is not probable, the water could have riſen high enough to run into the channel (at *e*, fig. 5.), and ſo communicate with the conductor before it reached the drain.

Cloſe to the chimney E, a dinner-bell hung in a common frame (*q*, fig. 2. and 4.). Three different perſons went up to examine this bell; but could not diſcover upon it any where the leaſt veſtige of the lightning.

Such are the facts we were able to collect from an aſſiduous examination of the Poor-houſe at Heckingham, and of thoſe witneſſes in the neighbourhood who knew any thing of the accident. We have ſtated the appearances as they preſented themſelves to us, with all the minuteneſs that could be preſerved without too much crowding the narrative, and independently of any opinions. Whether the earth or the clouds were poſitive at the time; whether the top or bottom of the hip was firſt affected by the ſtroke; whether all the lightning took its courſe through the hip, or part went that way, and part through the conductor; and how far the conductors were properly conſtructed, or adequately terminated; are queſtions which will naturally ſuggeſt themſelves to your conſideration.

It

It may be proper, however, to add the two following pieces of information.

One of the cripples in the Houſe of Induſtry, a middle-aged woman, aſſured us, that at the time of the accident, as ſhe was looking from the door of the hall (which is in the center of the front facing the ſouth), ſhe ſaw three balls of fire dart down; that one fell exactly oppoſite her; a ſecond ſeemed to ſtrike the corner of the houſe; and the other deſcended in the direction of a door in the eaſtern flank, which was not far out of the perpendicular line of the chimney B (ſee the general plan, fig. 1.). If any credit could be given to the teſtimony of ſuch a perſon in a matter like this, it would incline us to believe, that the exploſion was made in three ſtreams, of which one paſſed through the conductor of the chimney E, and another through the damaged corner of the houſe; whilſt the third fell on the ground, or, as the woman deſcribed it, on the great gate of the fore-court near the lodges (ſee the general plan, fig. 1.). We examined the gate and lodges, with the adjacent parts, but could no where diſcover any marks of injury; nor could we learn that any place in the neighbourhood had been ſtruck, or that any perſon, except this woman, pretended to have ſeen the courſe of the lightning.

In our return to town, through Norwich, we ſaw an ingenious gentleman of that city, who ſays, that he found the clouds negative there on the day of the accident at Heckingham. The two places are diſtant about eleven miles by the road.

It would be unpardonable to conclude this Report, without expreſſing our obligations to the Directors and Guardians of the Houſe of Induſtry at large, and to the neighbouring

Gentlemen in particular, for the liberal manner in which they ſeconded our endeavours to execute the commiſſion with which you had charged us. By their kind aſſiſtance proper workmen were provided; and every accommodation afforded us, that could contribute to the inveſtigation of this remarkable accident.

We have the honour to be,

GENTLEMEN,

Your moſt obedient humble ſervants,

C. BLAGDEN.
EDW. NAIRNE.

London, Feb. 7, 1782.

Fig. 1.

GENERAL PLAN.

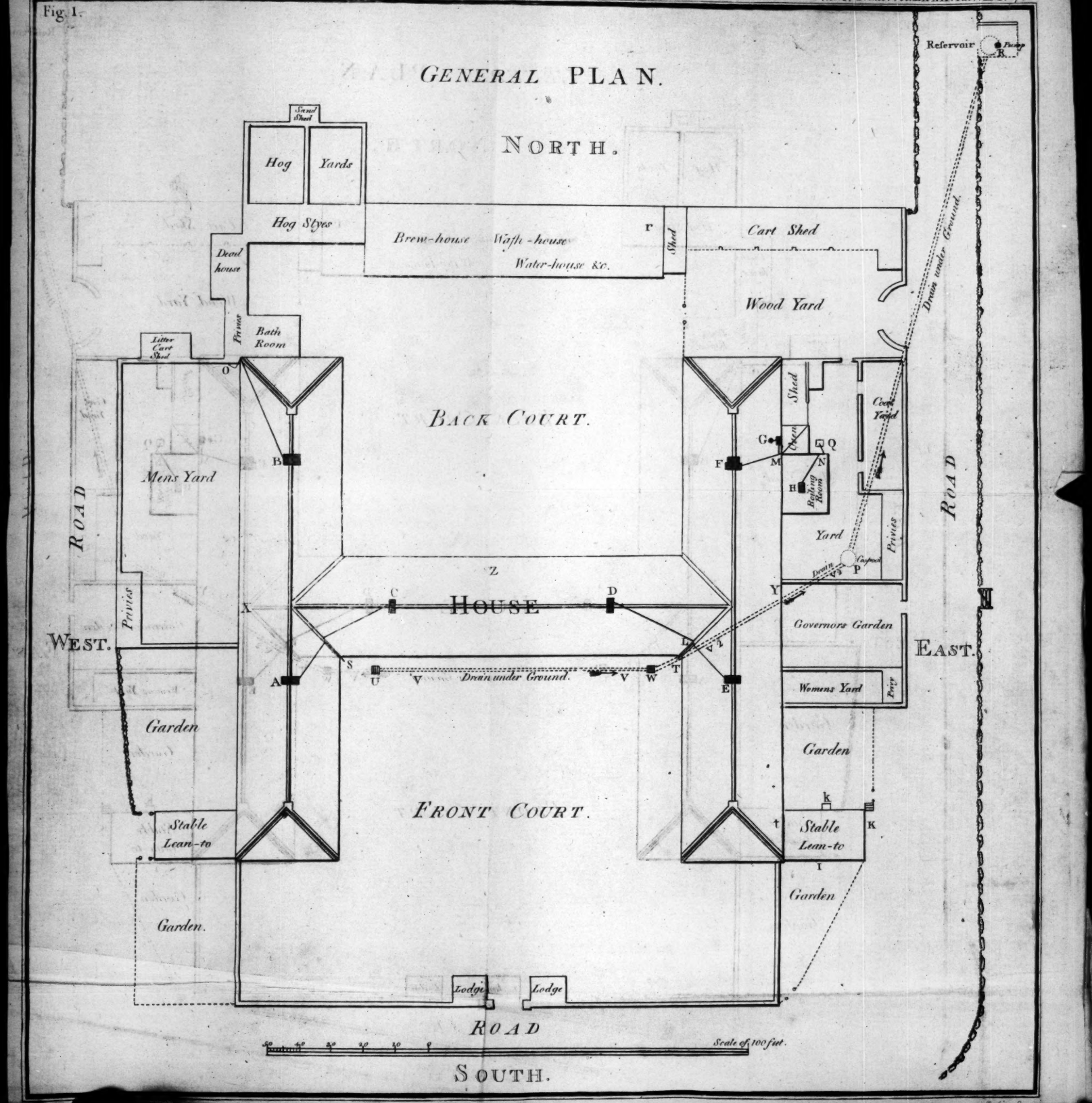


Fig. II.

SOUTH WEST VIEW of the EAST WING and part of the CENTER RANGE shewing the junction of the two Conductors that were nearest the Gable Lean-to.

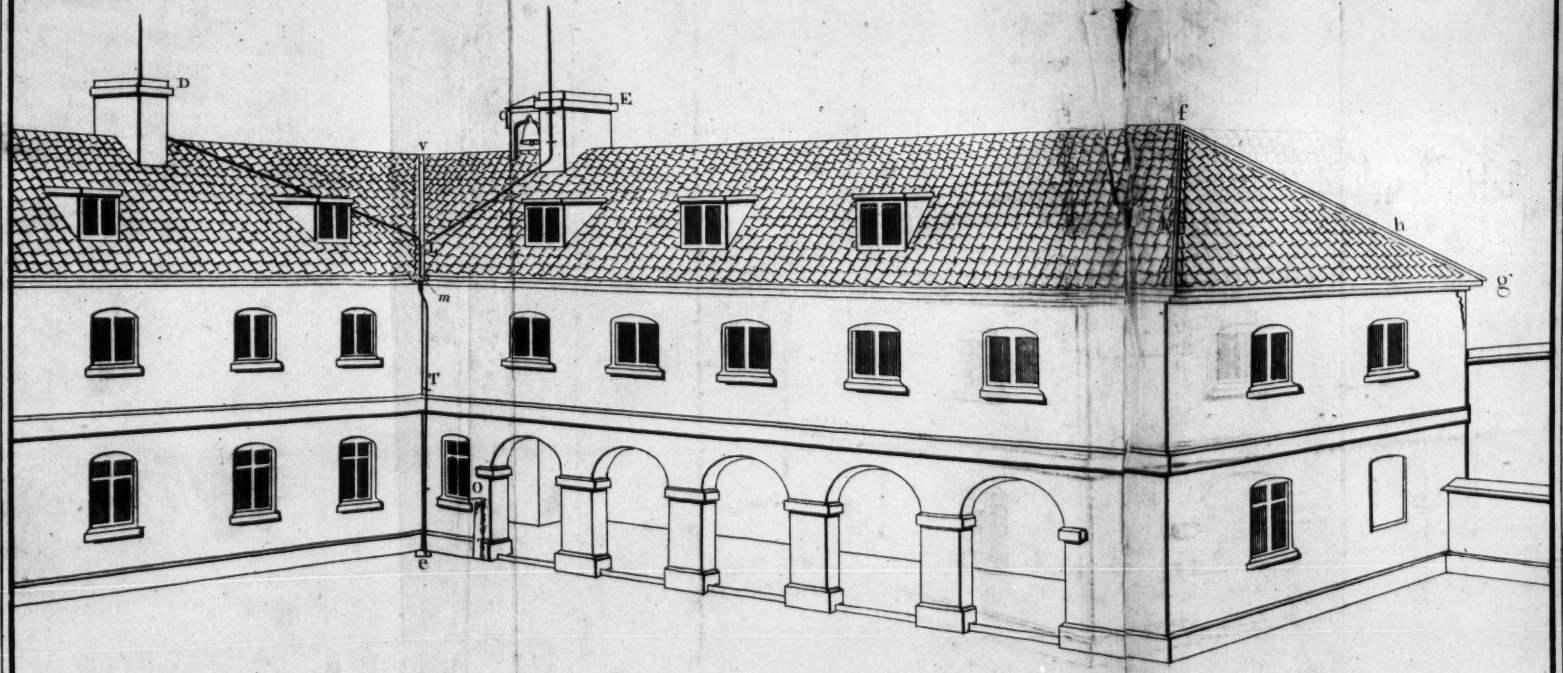


Fig. III.

ELEVATION *of the* SOUTH END *of* EAST WING, *with the Wall of the* LEAN-TO, *and part of the* SOUTH FRONT.

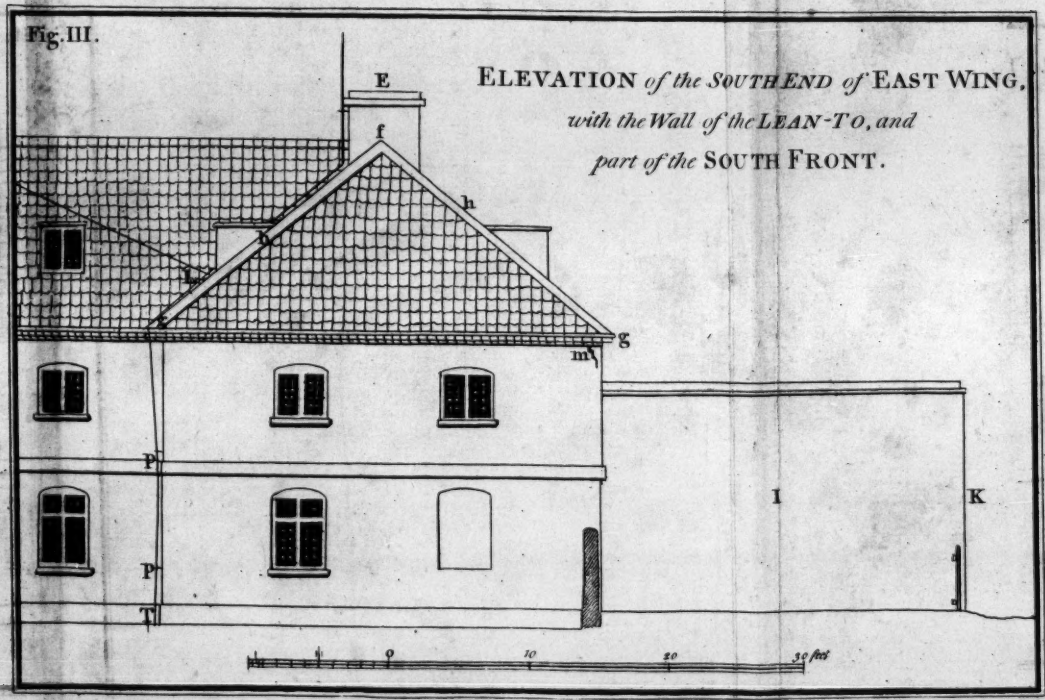


Fig. IV.

ELEVATION *of part of the* **EAST FLANK,**

with the adjoining **LEAN-TO.**

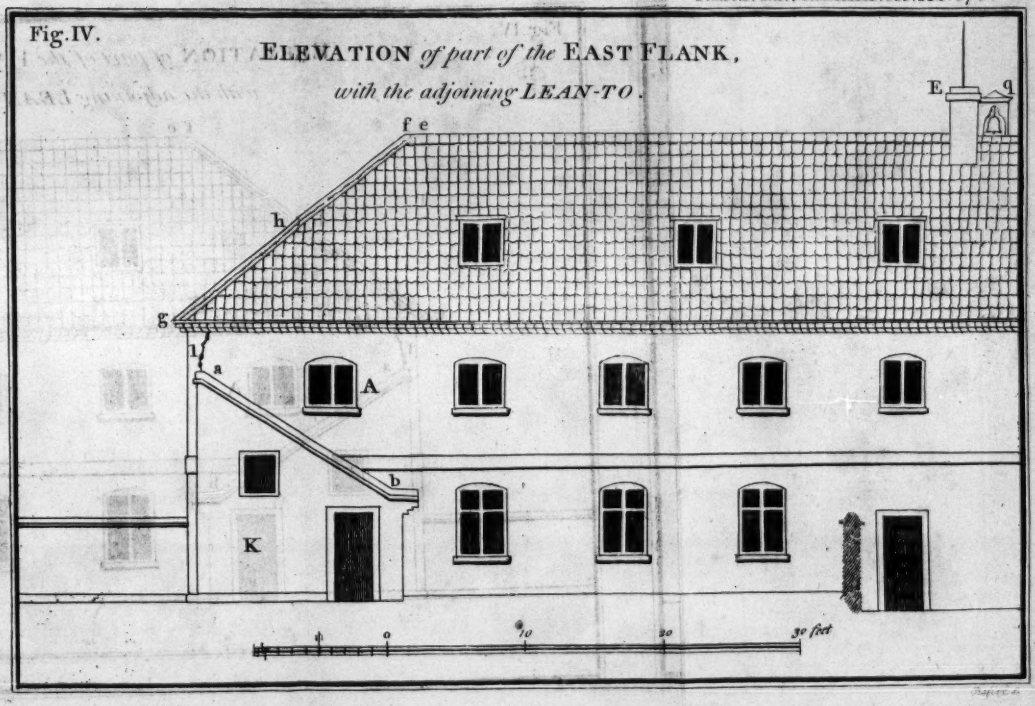
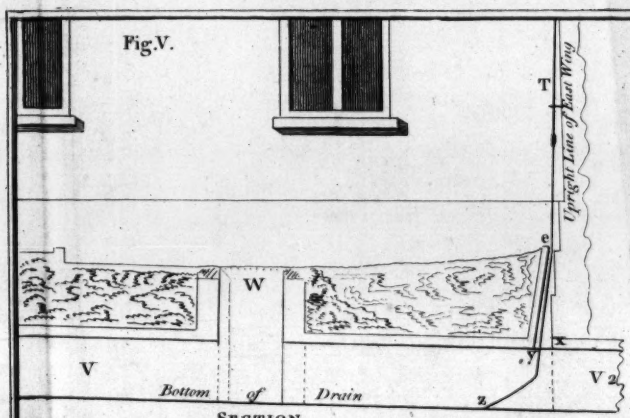


Fig.V.



SECTION

SECTIONS *and* PLAN *of the terminat*
of the Conductor at the South-East An
of the BUILDING.

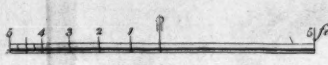


Fig.VI.

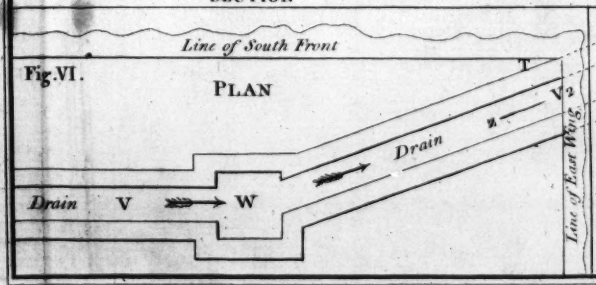
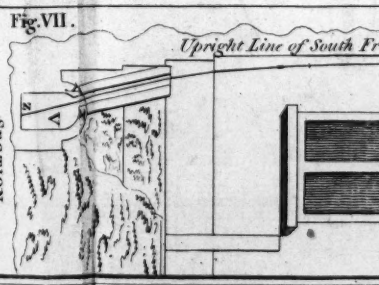


Fig.VII.



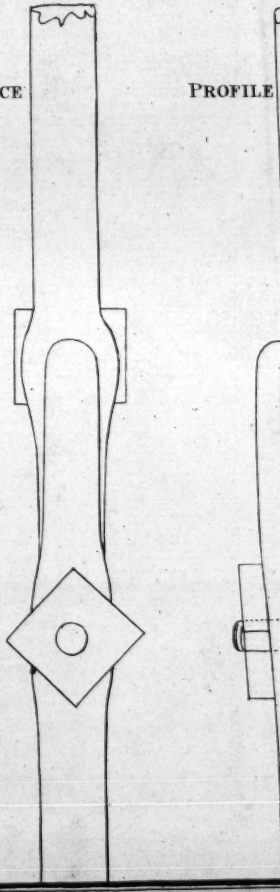
PERSPECTIVE VIEW
of the End of the Wall Plate and part of the Rafters.

Fig. XII.

FULL SIZE of the CONN
shewing the manner of their t

FACE

PROFILE



CONDUCTORS,
their being jointed.

FILE

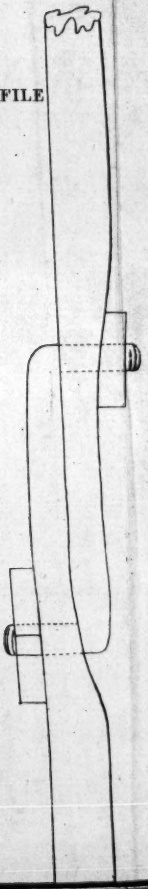
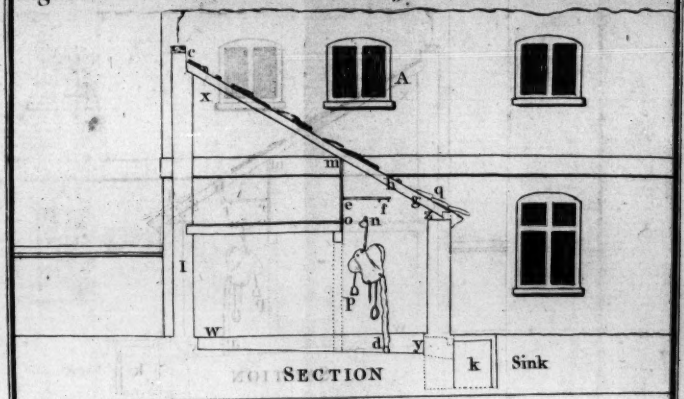


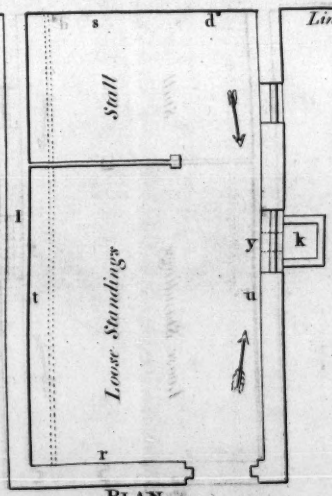
Fig. X. PLAN and SECTION of the Stable Lean-to.



SECTION

Fence Wall

Fig. IX.



PLAN

Fig. XI.

NORTH-EAST VIEW of the STAB
and part of the EAST FLAN

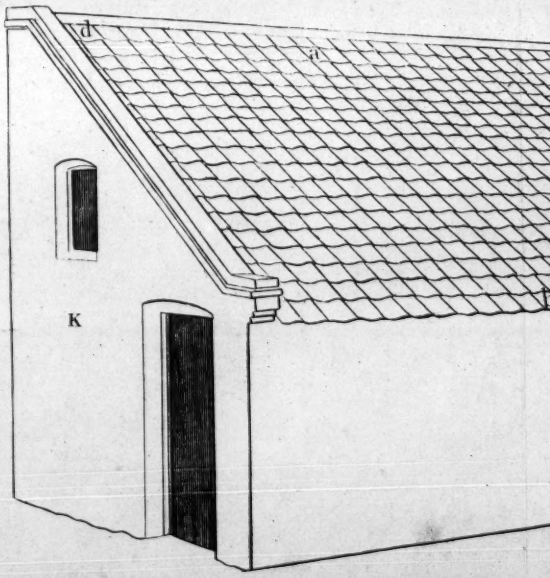
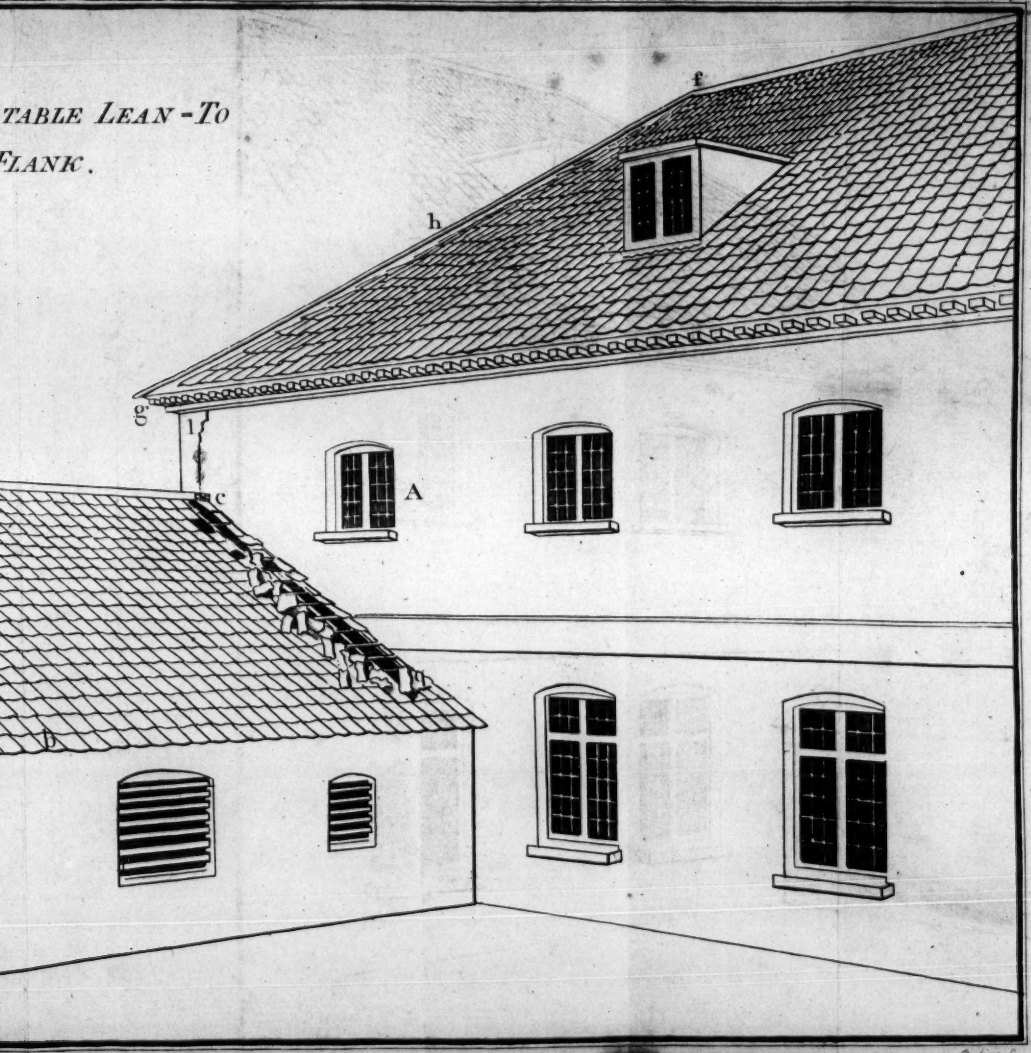


TABLE LEAN-TO
FLANK.



Basire S.

XXIII. *Account of the Organ of Hearing in Fiſh.* By John Hunter, *Eſq. F. R. S.*

Read Nov. 14, 1782.

NATURAL hiſtory has ever been conſidered as worthy the attention of the curious philoſopher, and therefore has in all ages kept pace with the other branches of knowledge; and as both arts and ſciences have, of late years, been cultivated to a degree, perhaps, beyond what was ever known before, we find alſo, that natural hiſtory has not been neglected; all Europe appears to be awake to it. In this iſland it has been purſued with more philoſophic ardour, than what was ever known in any country. It has become the ſtudy of men of independent fortunes, who not only ſpend their fortunes in the cultivation of this ſcience, but have riſqued their health and lives in purſuit of it, ſearching unknown regions to improve mankind, ſettling correſpondences every where, ſo as to bring in its materials into this country, in order to make it the ſchool of natural hiſtory. It is no wonder, then, that a ſpirit of inquiry is diffuſed through almoſt all ranks of men; and that though many cannot purſue it themſelves, yet they are eager to know what is already known, chuſing at leaſt to benefit by the induſtry of others.

Theſe reflections have induced me to trouble this learned Society with a ſhort account of the Organ of Hearing in

Fiſh, it being ſtill a ſubject of great diſpute, whether fiſh hear or not.

Some time between the years 1750 and 1760, I obſerved the organ of hearing in fiſh; and from that time to this, I only conſidered it as a link in the chain of the varieties in this ſenſe in different animals, in which there is a regular progreſſion, *viz.* from the moſt perfect animals down to the moſt imperfect poſſeſſed of this organ *.

As I do not intend to give, in this paper, a full account of this organ in any one fiſh, or of the varieties in different fiſh, but only of the organ in general; thoſe who may chuſe to purſue this part only of the animal œconomy may think it deficient in the deſcriptive parts. If it was a difficult taſk to expoſe this organ in fiſh, I ſhould perhaps be led to be more full in my deſcription of it, but there is nothing more eaſy than the expoſure of this organ in this animal in general.

As this paper is to be confined to this order of animals, I may be allowed juſt to obſerve here, that the claſs called ſepia has this organ alſo, but ſomewhat differently conſtructed from what it is in the fiſh.

The organs of hearing in this latter order of animals are placed on the ſides of the ſkull, or that cavity which contains the brain; but the ſkull itſelf makes no part of the organ, as it does in the quadruped and the bird. In ſome fiſh this organ is wholly ſurrounded by the parts compoſing this cavity, which in many is cartilaginous, the ſkeleton of theſe fiſh being

* Preparations to illuſtrate theſe facts have been ever ſince ſhewn in my collection to the curious both of this country and foreigners: when in ſhewing whatever was new, or ſuppoſed to be new, the ears of fiſh were always conſidered by me as one important article.

like

like thoſe of the ray kind; in others alſo, as in cod, ſalmon, &c. whoſe ſkeleton is bone, yet this part is cartilaginous.

In ſome fiſh this organ is in part within the cavity of the ſkull, or that cavity which alſo contains the brain, as in the ſalmon, cod, &c. the cavity of the ſkull projecting laterally, and forming a cavity there.

The organ of hearing in fiſh appears to grow in ſize with the animal, for its ſize is nearly in the ſame proportion with the ſize of the animal, which is not the caſe with the quadruped, &c. the organs being in them nearly as large in the growing fœtus as in the adult.

It is much more ſimple in fiſh than in all thoſe orders of animals who may be reckoned ſuperior, ſuch as quadrupeds, birds, and amphibious animals, but there is a regular gradation from the firſt to fiſh.

It varies in different orders of fiſh; but in all it conſiſts of three curved tubes, all of which unite with one another; this union forms in ſome only a canal, as in the cod, ſalmon, ling, &c.; and in others, a pretty large cavity as in the ray kind. In the jack there is an oblong bag, or blind proceſs, which is an addition to thoſe canals, and which communicates with them at their union. In the cod, &c. this union of the three tubes ſtands upon an oval cavity, and in the jack there are two of thoſe cavities; theſe additional cavities in theſe fiſh appear to anſwer the ſame purpoſe with the cavity in the ray or cartilaginous fiſh, which is the union of the three canals.

The whole is compoſed of a kind of cartilaginous ſubſtance, very hard or firm in ſome parts, and which in ſome fiſh is cruſted over with a thin bony lamella, ſo as not to allow them to collapſe; for as the ſkull does not form any part of

thoſe canals or cavities they muſt be compoſed of ſuch ſubſtance as is capable of keeping its form.

Each tube deſcribes more than a ſemi-circle. This reſembles in ſome reſpect what we find in moſt other animals, but differs in the parts being diſtinct from the ſkull *.

Two of the ſemi-circular canals are ſimilar to one another, may be called a pair, and are placed perpendicularly; the third is not ſo long; in ſome it is placed horizontally, uniting as it were the other two at their ends or terminations. In the ſkait it is ſomething different, being only united to one of the perpendiculars.

The two perpendiculars unite at one part in one canal, by one arm of each uniting, while the other two arms or horns have no connection with each other, and the arms of the horizontal unite with the other two arms of the perpendicular near the entrance into the common canal or cavity.

Near the union of thoſe canals into the common, they are ſwelled out into round bags, becoming there much larger.

In the ray kind they all terminate in one cavity, as has been obſerved; and in the cod they terminate in one canal, which in theſe fiſh is placed upon the additional cavity or cavities. In this cavity or cavities there is a bone or bones. In ſome there are two bones; as the jack has two cavities, we find in one of thoſe cavities two bones, and in the other only one; in the ray there is only a chalky ſubſtance †.

At this union of the two perpendiculars in ſome fiſh enters the external communication, or what may be called the external meatus. This is the caſe with all the ray kind, the external orifice

* The turtle and the crocodile have a ſtructure ſomewhat ſimilar to this; and the intention is the ſame, for their ſkulls make no part of the organ.

† This chalky ſubſtance is alſo found in the ears of amphibious animals.

of

of which is ſmall, and placed on the upper flat ſurface of the head; but it is not every genus or ſpecies of fiſh that has the external opening.

The nerves of the ear paſs outwards from the brain, and appear to terminate at once on the external ſurface of the ſwelling of the ſemi-circular tubes above deſcribed. They do not appear to paſs through thoſe tubes ſo as to get on the inſide, as is ſuppoſed to be the caſe in quadrupeds; I ſhould therefore very much ſuſpect, that the lining of thoſe tubes in the quadruped is not nerve, but a kind of internal perioſteum.

As it is evident that fiſh poſſeſs the organ of hearing, it becomes unneceſſary to make or relate any experiment made with live fiſh which only tends to prove this fact; but I will mention one experiment, to ſhew that ſounds affect them much, and is one of their guards, as it is in other animals. In the year 1762, when I was in Portugal, I obſerved in a nobleman's garden, near Liſbon, a ſmall fiſh-pond, full of different kinds of fiſh. Its bottom was level with the ground, and was made by forming a bank all round. There was a ſhrubbery cloſe to it. Whilſt I was laying on the bank, obſerving the fiſh ſwimming about, I deſired a gentleman, who was with me, to take a loaded gun, and go behind the ſhrubs and fire it. The reaſon for going behind the ſhrubs was, that there might not be the leaſt reflection of light. The inſtant the report was made, the fiſh appeared to be all of one mind, for they vaniſhed inſtantaneouſly into the mud at the bottom, raiſing as it were a cloud of mud. In about five minutes after they began to appear, till the whole came forth again.

XXIV. *Account of a new Electrometer.* *By Mr.* Abraham Brook; *communicated by Sir* Joſeph Banks, *Bart. P. R. S.*

Read May 30, 1782.

AAAAN, fig. 1. repreſents the electrometer in full ſize and proportion, ſtanding on a table, or the like. The foot B is a ſquare piece of board, 9¾ inches each way, reſting on three pins C, C, *c*, ſeen at the under ſide of the foot. C, C, with the broad heads, are ſcrews to ſet the inſtrument upright withal. D is a ſolid piece of glaſs, which ſupports and inſulates the inſtrument from the place on which it ſtands. The arms G1 and *g*, with the ball F, turn round on the wire H (which is ſolid braſs, as may be alſo the arm *g*), and, when in uſe, are put near at a right angle with G2 and H, being turned to the off ſide ſo as to be as much as poſſible out of each other's atmoſpheres or the atmoſphere of a jar, battery, prime conductor, &c. The arms G1 and G2 are hollow tubes of copper, not ſo heavy as wires. The balls I1, I2, are made of copper, and hollow, ſo as to be as light as poſſible. K repreſents a kind of face or dial plate to the inſtrument with its index, which is carried once round by the motion of the arm G2 with its ball I2 moving through a quarter, or 90 degrees, of a circle; this motion is given to it by the repulſive power of the charge, &c. of electricity between the two balls I2 and L. The ends of the index from its center are of different lengths. The longeſt end reaches to a graduated circle, divided into 90 equal parts, anſwering

anſwering to the 90°, which the arm G2 moves through. The ſhorteſt end reaches to a ſmaller circle, divided into 60 equal parts, anſwering to 60 grs. weight, or 60 diviſions, on the arm G1, with its ſliding weight *m*, each of which is equal to one grain, and the whole face is covered with a watch glaſs, to prevent the electricity from flying off at the points.

The top of the glaſs-ſupporter, or inſulator D, is cemented into a braſs cap M. This cap enters the ball L at bottom, and ſcrews into the upper part of the ball L at *a*. The top part of this cap M is tapered off to a cone about an inch and a half long or high. The lower end of the wire H has a hole made conically into it, ſo as to receive the upper part, or conical end, of the cap M, which permits all the upper part of the electrometer to turn round any way that may be neceſſary. The kind of ferrel O, with its baſe, is perforated for the lower end of the wire H to go through. The bent arm *b*, which ſupports the cup N, is ſcrewed into the baſe of the ferrel O, and turns freely round upon the wire H. The cup N is to receive the ball P of the arm, fig. 9. This arm ſhortens or lengthens, as may be wanted, by a wire ſliding into a tube. The end of the wire is ſlit, forming a ſpring in the tube to be ſteady. In this arm, fig. 9. is a kind of rule joint at *d*, that the arm may give way eaſily if wanted. The ſemi-circular end of the arm is a ſpring, and ſlips on to a ball from the prime-conductor, or the conductor itſelf (if they fit), jar, or battery. The ends of it are flat and broad, as repreſented in the drawing in miniature, of the electrometer at fig. 2. in the other drawing.

Fig. 2. to 11. ſhews the internal ſtructure of the electrometer.

Fig. 12. ſhews the part at *z* that ſcrews into the ball F, to ſupport the arm *g* with its ball *r*. This piece, which is made

hollow on the ſide next the wire H, ſo as to fit, and is ſcrewed in, ſo as to preſs againſt H, ſerves as a ſpring to keep the ball F ſteady, which ſlides up and down, as well as turns round, on H.

In order to make the diviſions of G1, fig. 1. exactly a grain each, firſt ſlide the weight *m* towards the ball F, fig. 1. till it is an exact counter-balance to the weight in F. At one end of the weight *m* let the diviſions begin; then ſuſpend any tolerable pair of ſcales, ſo that the bottom of one of them may reſt on the top of the ball *r*; then lay the ball I1 into the ſcale, and ſlide the weight *m* near to I1, and put as many grains into the other ſcale as will juſt raiſe the ball I1 in the ſcale; then mark the arm G1 at the ſame end of the weight *m*, and divide the ſpace between the two marks into as many parts as there are grains in the ſcale, which may be divided and ſub-divided into halves and quarters.

The arm G2 being repelled ſhews when the charge is increaſing, &c.; and I1 tells what ſuch a repulſive power is between two balls of the ſize of theſe in grains, according to the number the weight *m* reſts at when lifted up by the repulſive power of a charge. The longeſt end of the index K ſhews how many degrees of a circle G2 is repelled; and, by many trials, according to the number of grains, the arm G1 ſhews, when it is lifted up, and the weight *m* put at different places, ſuch reſpective numbers of grains may be marked on the leaſt graduated circle on the dial plate where the ſhorteſt end of the index points; ſo that when all the grains are thus marked on the dial plate, thus aſcertained by the arm G1, all theſe parts of the inſtrument, that is, the ball F with the arms G1 and *g* may be taken off, and the inſtrument is then graduated to be uſed without them; but I do not know how the grains can be ſo

exactly

exactly marked and ascertained as by these parts being on the instrument: nor do I mean to confine the number of grains or divisions on G1; but, I think, my experience seems to tell me, that no glass to be *charged*, as we call it, with electricity, will bear a greater charge than that whose repulsive force, between two balls of this size, equals 60 grains weight, before it will be perforated or struck through. Nay, I have not found many instances where it would stand 50 grains; and, I think, it is very hazardous to go more than 45 grains.

Thus, by knowing the quantity of coated surface, and the diameter of the balls, as I1 and *r*, I would say, so much or so much coated surface charged to so many or so many grains repulsion between two balls of such or such a size would melt a wire of this or that size, or do such a thing, kill such an animal, &c.; and if balls, wires, or arms of this size, are found too small, larger may be made on the same plan.

In respect to the advantages of this electrometer above *those now in use*, I do not, perhaps, know them all; and lest my partiality may prejudice me in behalf of my own contrivance, would rather leave them to the judgement of others; my opinion however is, that all that I have seen or heard of are such as speak no intelligible language, and that this speaks so as to be understood universally; for, unless the repulsive power of the charge of different glasses be very different, this electrometer, or any other electrometer, made after this manner, must, I should think, speak very nearly the same language, it being known how much coated surface there is, and the size of the balls; but if the size of the balls be not the same, the language the instrument speaks will be very different. Although other Electrometers shewed a

greater or leſſer charge or power, by an arm being repelled to a greater or leſſer diſtance, or by ſtriking differently at different diſtances, yet the power of the charge was not in any manner aſcertained; we could ſay, that the arm or index was repelled to ſuch or ſuch a number of degrees of a circle, or that it ſtruck to ſuch or ſuch a diſtance; but the repulſive power of a charge to repel the index ſo much, or ſo many degrees of a circle, or the ſtrength of the charge to ſtrike to ſuch a diſtance was not (that I know of) in any manner intelligibly aſcertained. This ſhews it by the weight that the repulſive power has to lift up in grains, &c.; which weight is to be proved by any tolerable pair of ſcales and weights; and I do not know any other method that has been yet tried to ſhew the different ſtrength of charges ſo good as that of repulſion.

All the neceſſary parts of the inſtrument being made of metal and glaſs that is pretty ſtout, I think, the electricity is leſs liable to eſcape than by wood, &c. I have tried reeds on account of their being light, and covered them with tin-foil, or gilded them to make them good conductors; but ſo frequently found inconveniencies from them by points riſing up, the celerity of moving, and the different weight of them at different times owing to moiſture, change of the weather, and the like, that I have laid them all aſide, and find my preſent inſtrument as free from theſe inconveniencies as I could expect; nor is it liable to be out of order, if proper care is taken of it.

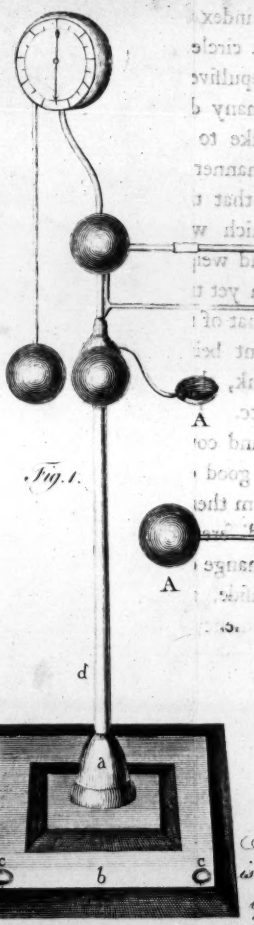
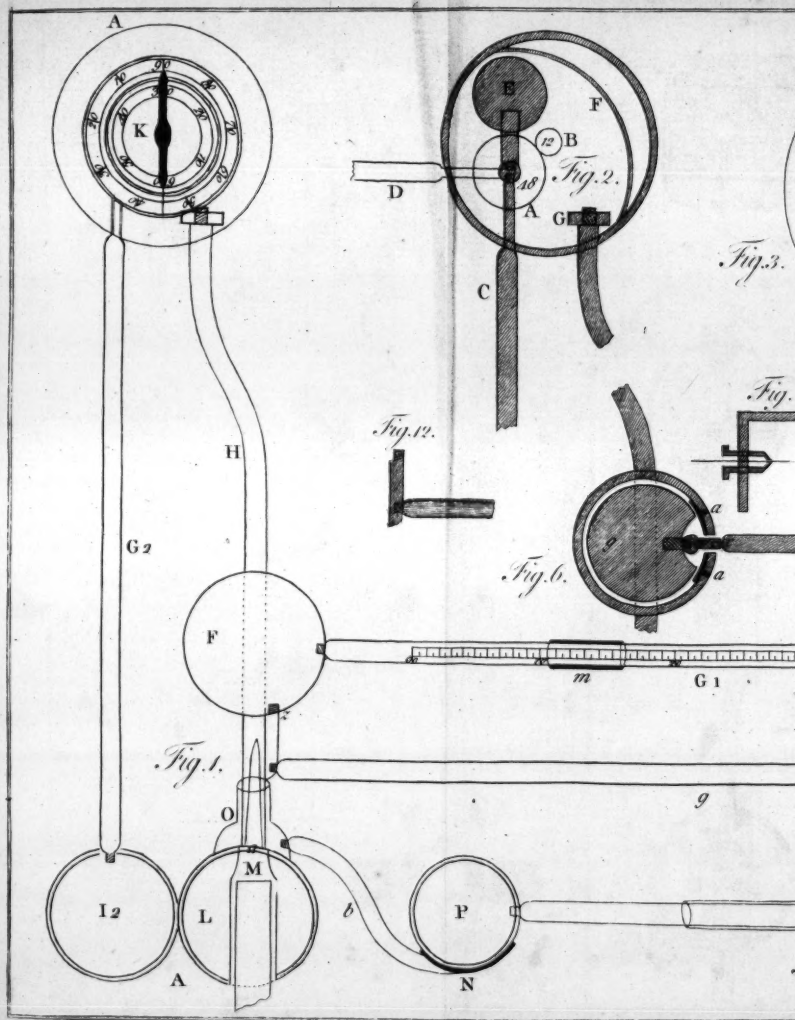


Fig. 1.

Fig. 2.

Fig. 1. represents the Electrometer in miniature. Fig. 2. is an Arm to make a communication from a Battery, &c. by the Semicircular End being slipt on to a Ball of the Battery &c. and the Ball A being laid into the Cup A. Fig. 1.


A
K
H
G 2
F
Fig. 1.
O
M
I 2
L
A
b
P
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E
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D
A
G
C
Fig. 2.
Fig. 3.
Fig. 12.
Fig. 6.
a
a
m
G 1
g

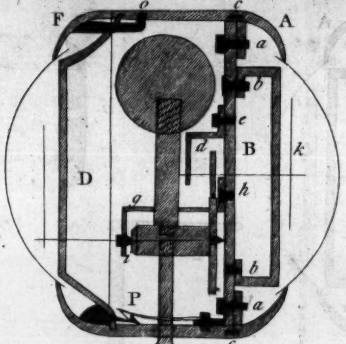

Fig. 3.

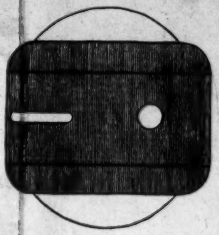

Fig. 5.

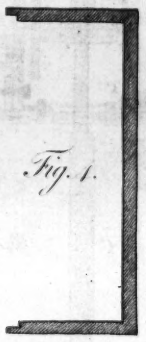

Fig. 1.

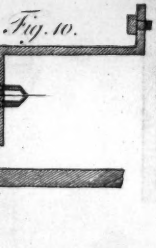

Fig. 10.

Fig. 7.

Fig. 11.

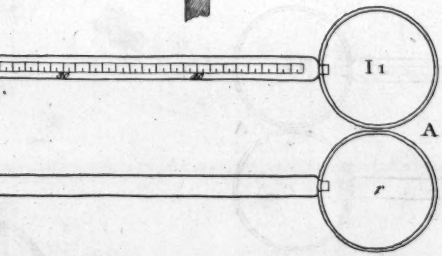

Fig. 8.

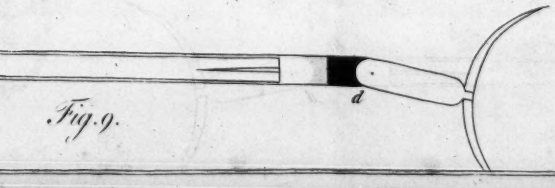

Fig. 9.

XXV. *A new Method of inveſtigating the Sums of infinite Series. By the Rev.* S. VINCE, *A. M. of* CAMBRIDGE, *in a Letter to* Henry Maty, *A. M. Secretary.*

Read June 6, 1782.

SIR,

HAVING lately diſcovered ſome very eaſy methods of inveſtigating the ſums of certain infinite ſeries, I have taken the liberty of requeſting the favour of you to preſent them to the Royal Society. I have divided the ſubject into three parts: the firſt contains a new and general method of finding the ſum of thoſe ſeries which DE MOIVRE has found in one or two particular caſes; but whoſe method, although it be in appearance general, will, upon trial, be found to be abſolutely impracticable. The ſecond contains the ſummation of certain ſeries, the laſt differences of whoſe numerators become equal to nothing. The third contains obſervations on a *correction* which is neceſſary in inveſtigating the ſums of certain ſeries by collecting two terms into one, with its application to a variety of caſes.

I am, &c.

Cambridge,
May 3, 1782.

PART

PART I.

LEMMA I.

Let r be any whole number, and then the fluent of $\frac{\dot{x}}{1+x^r}$ *can always be exhibited by circular arcs and logarithms; but when* $x=1$, *the fluent of the same fluxion will be expressed by the infinite series* $1-\frac{1}{r+1}+\frac{1}{2r+1}-\frac{1}{3r+1}+$ &c. *the sum of this series therefore can always be found by circular arcs and logarithms.*

LEMMA II.

To find the sum of the infinite series $\frac{a}{1\cdot r+1}+\frac{a+b}{r+1\cdot 2r+1}+\frac{a+2b}{2r+1\cdot 3r+1}+$ &c.

Assume $1-\frac{1}{r+1}+\frac{1}{2r+1}-\frac{1}{3r+1}+$ &c. $=S$; therefore,

$$(A)\ 1-\frac{1}{1\cdot r+1}+\frac{r+1}{r+1\cdot 2r+1}-\frac{2r+1}{2r+1\cdot 3r+1}+\text{&c.}\ldots=S.$$

In the first series, add together the 1st and 2d, the 2d and 3d, &c. &c. terms, and the resulting series will evidently be equal to twice that series *minus* the first term; therefore,

$$(B),\ \frac{r}{1\cdot r+1}-\frac{r}{r+1\cdot 2r+1}+\frac{r}{2r+1\cdot 3r+1}-\text{&c.}\ldots=2S-1.$$

$$\text{Now } \left(\frac{A}{r}\right)\ \frac{1}{r}-\frac{\frac{1}{r}}{1\cdot r+1}+\frac{1+\frac{1}{r}}{r+1\cdot 2r+1}-\frac{2+\frac{1}{r}}{2r+1\cdot 3r+1}+\text{&c.}\ldots=\frac{S}{r},$$

$$\text{or } \left.\begin{aligned}&\frac{1}{r+1\cdot 2r+1}-\frac{2}{2r+1\cdot 3r+1}+\text{&c.}\ldots\\ &+\frac{1}{r}-\frac{\frac{1}{r}}{1\cdot r+1}+\frac{\frac{1}{r}}{r+1\cdot 2r+1}-\text{&c.}\ldots\end{aligned}\right\}=\frac{S}{r};$$

Now

Now the ſum of the lower ſeries, omitting the firſt term, is equal to $-$ B divided by r^2, or $= -\frac{2S-1}{r^2}$; hence, by tranſpoſition, and, multiplying both ſides by b, we ſhall have,

$\frac{b}{r+1 \cdot 2r+1} - \frac{2b}{2r+1 \cdot 3r+1} + \&c. \ldots = \frac{bS}{r} + \frac{2bS - \overline{r+1} \cdot b}{r^2}$; alſo by multiplying B by $\frac{a}{r}$ we have

$\frac{a}{1 \cdot r+1} - \frac{a}{r+1 \cdot 2r+1} + \frac{a}{2r+1 \cdot 3r+1} - \&c. \ldots = \frac{2aS-a}{r}$; ſubtract the laſt equation but one from the laſt, and we ſhall have

$\frac{a}{1 \cdot r+1} - \frac{a+b}{r+1 \cdot 2r+1} + \frac{a+2b}{2r+1 \cdot 3r+1} - \&c. \ldots = \frac{\overline{2ra-r+2} \cdot b \times S - ra + \overline{r+1} \cdot b}{r^2}$.

Cor. 1. Hence it appears, that the ſum of this ſeries can never be exhibited in finite terms, except $a : b$ as $r+2 : 2r$, in which caſe the ſum is equal to $\frac{a}{r+2}$.

Hence, if $a=3, b=2$, then $r=1$; $\therefore \frac{3}{1 \cdot 2} - \frac{5}{2 \cdot 3} + \frac{7}{3 \cdot 4} - \&c. \ldots = 1$;

if $a=1, b=4$, then $r=\frac{4}{3}$; $\therefore \frac{5}{3 \cdot 7} - \frac{9}{7 \cdot 11} + \frac{13}{11 \cdot 15} - \frac{17}{15 \cdot 19} + \&c. \ldots = \frac{1}{6}$;

if $a=4, b=3$, then $r=\frac{6}{5}$; $\therefore \frac{4}{5 \cdot 11} - \frac{7}{11 \cdot 17} + \frac{10}{17 \cdot 23} - \frac{13}{23 \cdot 29} + \&c. \ldots = \frac{1}{20}$;

Cor 2. Put $a=c-b$, and we ſhall have, after tranſpoſition,

$\frac{c}{r+1 \cdot 2r+1} - \frac{c+b}{2r+1 \cdot 3r+1} + \&c. \ldots = \frac{\overline{3r+2} \cdot b - 2rc \times S - \overline{2r+1} \quad b+rc}{r^2} \div \frac{c-b}{r+1}$.

PROP. I.

To find the ſum of the infinite ſeries $\frac{m}{1 \cdot r+1 \cdot 2r+1} +$

$\frac{m+n}{2r+1 \cdot 3r+1 \cdot 4r+1} - \frac{m+2n}{4r+1 \cdot 5r+1 \cdot 6r+1} + \&c.$

Every

Every ſeries of this kind may be reſolved into the following ſeries $\frac{a}{1\cdot r+1}-\frac{a+b}{r+1\cdot 2r+1}+\frac{a+2b}{2r+1\cdot 3r+1}-\frac{a+3b}{3r+1\cdot 4r+1}+$ &c. for if we reduce two terms of this ſeries into one, it will become

$$\frac{2ar-b}{1\cdot r+1\cdot 2r+1}+\frac{2ra+\overline{2r-1}\cdot b}{2r+1\cdot 3r+1\cdot 4r+1}+\frac{2ra+\overline{4r-1}\cdot b}{4r+1\cdot 5r+1\cdot 6r+1}+\text{ &c.}$$

where the denominators being the ſame as in the given ſeries, and the numerators alſo in arithmetic progreſſion, we have only to take a and b ſuch quantities that the reſpective numerators may be alſo equal; aſſume, therefore, $2ra-b=m$, $2ra+\overline{2r-1}\cdot b=m+n$; therefore, $b=\frac{n}{2r}$, $a=\frac{2rm+n}{4r^2}$, which ſubſtituted for a and b in LEM. 2. gives

$$\frac{m}{1\cdot r+1\cdot 2r+1}+\frac{m+n}{2r+1\cdot 3r+1\cdot 4r+1}+\frac{m+2n}{4r+1\cdot 5r+1\cdot 6r+1}+\text{ &c.}\ldots$$
$$=\frac{2rm-r+1\cdot n}{2r}\times S+\frac{\overline{2r+1}\cdot n-2rm}{4r^3}.$$

Let $r=1$, and we have

$$\frac{m}{1\cdot 2\cdot 3}+\frac{m+n}{3\cdot 4\cdot 5}+\frac{m+2n}{5\cdot 6\cdot 7}+\text{ &c.}\ldots=\frac{\overline{m-n}}{4}\cdot S+\frac{3n-2m}{4}.$$

If $m=1$, $n=3$, $\frac{1}{1\cdot 2\cdot 3}+\frac{4}{3\cdot 4\cdot 5}+\frac{7}{5\cdot 6\cdot 7}+\text{ &c.}\ldots=\frac{7}{4}-2S$;

$m=1$, $n=0$, $\frac{1}{1\cdot 2\cdot 3}+\frac{1}{3\cdot 4\cdot 5}+\frac{1}{5\cdot 6\cdot 7}+\text{ &c.}\ldots=S-\frac{1}{2}$.

Let $r=2$, and the ſeries becomes

$$\frac{m}{1\cdot 3\cdot 5}+\frac{m+n}{5\cdot 7\cdot 9}+\frac{m+2n}{9\cdot 11\cdot 13}+\text{ &c.}\ldots=\frac{4m-3n}{16}\times S+\frac{5n-4m}{32}.$$

If $m=1$, $n=1$, $\frac{1}{1\cdot 3\cdot 5}+\frac{2}{5\cdot 7\cdot 9}+\frac{3}{9\cdot 11\cdot 13}+\text{ &c.}\ldots=\frac{S}{16}+\frac{1}{32}$;

$m=1$, $n=0$, $\frac{1}{1\cdot 3\cdot 5}+\frac{1}{5\cdot 7\cdot 9}+\frac{1}{9\cdot 11\cdot 13}+\text{ &c.}\ldots=\frac{S}{4}-\frac{1}{8}$.

Let $r=5$, and we ſhall have

$$\frac{m}{1\cdot 6\cdot 11}+\frac{m+n}{11\cdot 16\cdot 21}+\frac{m+2n}{21\cdot 26\cdot 31}+\text{ &c.}\ldots=\frac{5m-3n}{125}\times S+\frac{11n-10m}{500}.$$

If $m=1, n=1$, $\frac{1}{1.6.11}+\frac{2}{11.16.21}+\frac{3}{21.26.31}+\&c.\ldots=\frac{2}{125}\times S+\frac{1}{500}$;

$m=1, n=0$, $\frac{1}{1.6.11}+\frac{1}{11.16.21}+\frac{1}{21.26.31}+\&c.\ldots=\frac{8}{25}-\frac{1}{50}$.

Cor. If $2r:r+1::n:m$, the ſum of the ſeries can be accurately found, and will be equal to $\frac{m}{2r.r+1}$. Let therefore $m=r+1$, and then $n=2r$, conſequently

$$\frac{1}{1.2r+1}+\frac{1}{2r+1.4r+1}+\frac{1}{4r+1.6r+1}+\&c.\ldots=\frac{1}{2r};$$

which is alſo known from other principles.

PROP. II.

To find the ſum of the infinite ſeries $\frac{m}{r+1.2r+1.3r+1}+\frac{m+n}{3r+1.4r+1.5r+1}+\frac{m+2n}{5r+1.6r+1.7r+1}+\&c.$

This ſeries reſolves itſelf into

$$\frac{c}{r+1.2r+1}-\frac{c+b}{2r+1.3r+1}+\frac{c+2b}{3r+1.4r+1}-\&c.;$$

for by reduction, as before, it becomes

$$\frac{2cr-\overline{r+1}.b}{r+1.2r+1.3r+1}+\frac{2cr+\overline{r-1}.b}{3r+1.4r+1.5r+1}+\frac{2cr+\overline{3r-1}.b}{5r+1.6r+1.7r+1}+\&c.$$

where the denominators are the ſame as in the given ſeries, and the numerators in arithmetic progreſſion; aſſume therefore $2cr-\overline{r+1}.b=m$, $2cr+\overline{r-1}.b=m+n$, hence $b=\frac{n}{2r}$, $c=\frac{2rm+\overline{r+1}.n}{4r^2}$, which, ſubſtituted in cor. 2. LEM. 2. give

$$\frac{m}{r+1.2r+1.3r+1}+\frac{m+n}{3r+1.4r+1.5r+1}+\frac{m+3n}{5r+1.6r+1.7r+1}+\&c.\ldots$$
$$=\frac{2r+1.n-2rm}{2r^3}\times S+\frac{2rm-\overline{3r+1}.n}{4r^3}+\frac{2rm+\overline{r-1}.n}{4r^3.r+1}.$$

Cor. 1. In prop. 1. ſubſtitute a for m, and $2b$ for n, and we have

$$\frac{a}{1\cdot r+1\cdot 2r+1}+\frac{a+2b}{2r+1\cdot 3r+1\cdot 4r+1}+\frac{a+4b}{4r+1\cdot 5r+1\cdot 6r+1}+\&c.\ldots.=$$
$$\frac{ra-r+1\cdot b}{r^3}\times S+\frac{2r+1\cdot b-ra}{2r^3}.$$

Alſo in this prop. ſubſtitute $a+b$ for m, and $2b$ for n, and we have

$$\frac{a+b}{r+1\cdot 2r+1\cdot 3r+1}+\frac{a+3b}{3r+1\cdot 4r+1\cdot 5r+1}+\&c.\ldots.=$$
$$\frac{r+1\cdot b-ra}{r^3}\times S+\frac{ra-2r+1\cdot b}{2r^3}+\frac{ra+b}{2r^2\times r+1}.$$

Subtract this latter ſeries from the former, and

$$\frac{a}{1\cdot r+1\cdot 2r+1}-\frac{a+b}{r+1\cdot 2r+1\cdot 3r+1}+\frac{a+2b}{2r+1\cdot 3r+1\cdot 4r+1}-\&c.\ldots.=$$
$$\frac{2ra-r+1\cdot 2b}{r^3}\times S+\frac{2r+1\cdot b-ra}{r^3}-\frac{ra+2b}{2r^2\times r+1}.$$

Let $r=1$, and we have

$$\frac{a}{1\cdot 2\cdot 3}-\frac{a+b}{2\cdot 3\cdot 4}+\frac{a+2b}{3\cdot 4\cdot 5}-\&c.\ldots.=\frac{2a-4b}{1}\times S+\frac{11b-5a}{4}.$$

If $a=1$, $b=0$, $\frac{1}{1\cdot 2\cdot 3}-\frac{1}{2\cdot 3\cdot 4}+\frac{1}{3\cdot 4\cdot 5}-\&c.\ldots.=2S-\frac{5}{4}$;

$a=1$, $b=2$, $\frac{1}{1\cdot 2\cdot 3}-\frac{3}{2\cdot 3\cdot 4}+\frac{5}{3\cdot 4\cdot 5}-\&c.\ldots.=\frac{17}{4}-6S$.

Let $r=3$, and we have

$$\frac{a}{1\cdot 4\cdot 7}-\frac{a+b}{4\cdot 7\cdot 10}+\frac{a+2b}{7\cdot 10\cdot 13}-\&c.\ldots.=\frac{6a-8b}{27}\times S+\frac{53b-33a}{216}.$$

If $a=1$, $b=0$, $\frac{1}{1\cdot 4\cdot 7}-\frac{1}{4\cdot 7\cdot 10}+\frac{1}{7\cdot 10\cdot 13}-\&c.\ldots.=\frac{2}{9}S-\frac{11}{72}$;

$a=1$, $b=1$, $\frac{1}{1\cdot 4\cdot 7}-\frac{2}{4\cdot 7\cdot 10}+\frac{3}{7\cdot 10\cdot 13}-\&c.\ldots.=\frac{5}{54}-\frac{2}{27}S$.

If, inſtead of ſubſtituting in prop. 1. $2b$ and a for n and m, we had ſubſtituted two other quantities, as $2r$ and s, and then proceeded as above, a ſeries would have been formed, the numerators of whoſe alternate terms would have formed each a a ſeparate arithmetic progreſſion.

2 If

If the latter ſeries had been *added* to the former a ſeries would have been formed whoſe terms would have been all poſitive; but as I purpoſe, in the ſecond part of this paper, to give a general method of ſumming all ſuch ſeries, I ſhall not ſtop here to apply this method of inveſtigation.

Cor. 2. In propoſition 2. ſubſtitute a for m, and $2b$ for n, and we ſhall have

$$\frac{a}{r+1\,.\,2r+1\,.\,3r+1}+\frac{a+2b}{3r+1\,.\,4r+1\,.\,5r+1}+\&c.\ldots=$$
$$\frac{2r+1\,.\,b-ra}{r^3}\times S+\frac{ra-3r+1\,.\,b}{2r^3}+\frac{ra-r-1\,.\,b}{2r^2\,.\,r+1}.$$

Alſo in prop. 1. write $a-b$ for m, and $2b$ for n, and there reſults

$$\frac{a+b}{2r+1\,.\,3r+1\,.\,4r+1}+\frac{a+3b}{4r+1\,.\,5r+1\,.\,6r+1}+\&c.\ldots=$$
$$\frac{ra-2r+1\,.\,b}{r^3}\times S+\frac{r+1\,.\,b-ra}{2r^3}-\frac{a-b}{r+1\,.\,2r+1}.$$

Subtract this latter ſeries from the former, and we ſhall have

$$\frac{a}{r+1\,.\,2r+1\,.\,3r+1}-\frac{a+b}{2r+1\,.\,3r+1\,.\,4r+1}+\frac{a+2b}{3r+1\,.\,4r+1\,.\,5r+1}-\&c.\ldots$$
$$=\frac{2r+1\,.\,2b-2ra}{r^3}\times S+\frac{ra-3r+1\,.\,b}{r^3}+\frac{ra-r-1\,.\,b}{2r^2\,.\,r+1}+\frac{a-b}{r+1\,.\,2r+1}.$$

PROP. III.

To find the ſum of the infinite ſeries $\frac{m}{1\,.\,r+1\,.\,2r+1\,.\,3r+1}+$
$$\frac{m+n}{2r+1\,.\,3r+1\,.\,4r+1\,.\,5r+1}+\frac{m+2n}{4r+1\,.\,5r+1\,.\,6r+1\,.\,7r+1}+\&c.$$

This ſeries reſolves itſelf into

$$\frac{a}{1\,.\,r+1\,.\,2r+1}-\frac{a+b}{r+1\,.\,2r+1\,.\,3r+1}+\frac{a+2b}{2r+1\,.\,3r+1\,.\,4r+1}-\&c.$$

for by reduction it becomes

$$\frac{3ra-b}{1\cdot r+1\cdot 2r+1\cdot 3r+1}+\frac{3ar+4r-1\cdot b}{2r+1\cdot 3r+1\cdot 4r+1\cdot 5r+1}+$$

$$\frac{3ar+8r-1\cdot b}{4r+1\cdot 5r+1\cdot 6r+1\cdot 7r+1}+\&c.$$

where the numerators are in arithmetic progression, and the denominators the same as in the given series; assume therefore $3ra-b=m$, $3ra+\overline{4r-1}\cdot b=m+n$, hence $b=\frac{n}{4r}$, $a=\frac{4rm+n}{12r^2}$; substitute these values into cor. 1. prop. 2. and we have

$$\frac{m}{1\cdot r+1\cdot 2r+1\cdot 3r+1}+\frac{m+n}{2r+1\cdot 3r+1\cdot 4r+1\cdot 5r+1}+$$

$$\frac{m+2n}{4r+1\cdot 5r+1\cdot 6r+1\cdot 7r+1}+\&c.\ldots=\frac{4rm-3\overline{r+2}\cdot n}{6r^2}\times S+$$

$$\frac{3r+1\cdot n-2rm}{6r^2}-\frac{rm+n}{6r^2\cdot r+1}.$$

Let $r=1$, and we have

$$\frac{m}{1\cdot 2\cdot 3\cdot 4}+\frac{m+n}{3\cdot 4\cdot 5\cdot 6}+\frac{m+2n}{5\cdot 6\cdot 7\cdot 8}+\&c.\ldots=\frac{4m-5n}{6}\times S+\frac{7n-5m}{12};$$

If $m=1, n=1$, $\frac{1}{1\cdot 2\cdot 3\cdot 4}+\frac{1}{3\cdot 4\cdot 5\cdot 6}+\frac{1}{5\cdot 6\cdot 7\cdot 8}+\&c.\ldots=\frac{2}{3}S-\frac{5}{12}$;

$m=1, n=3$, $\frac{1}{1\cdot 2\cdot 3\cdot 4}+\frac{2}{3\cdot 4\cdot 5\cdot 6}+\frac{2}{5\cdot 6\cdot 7\cdot 8}+\&c.\ldots=\frac{1}{6}-\frac{1}{6}S$;

$m=7, n=5$, $\frac{7}{1\cdot 2\cdot 3\cdot 4}+\frac{12}{3\cdot 4\cdot 5\cdot 6}+\frac{17}{5\cdot 6\cdot 7\cdot 8}+\&c.\ldots=\frac{1}{2}S$;

Let $r=2$, and we have

$$\frac{m}{1\cdot 3\cdot 5\cdot 7}+\frac{m+n}{5\cdot 7\cdot 9\cdot 11}+\frac{m2n}{9\cdot 11\cdot 13\cdot 15}+\&c.\ldots=\frac{m-n}{12}\times S-\frac{19n-16m}{288}.$$

If $m=1, n=3$, $\frac{1}{1\cdot 3\cdot 5\cdot 7}+\frac{4}{5\cdot 7\cdot 9\cdot 11}+\frac{7}{9\cdot 11\cdot 13\cdot 15}+\&c.\ldots=\frac{41}{288}-\frac{1}{6}S$;

$m=1, n=0$, $\frac{1}{1\cdot 3\cdot 5\cdot 7}+\frac{1}{5\cdot 7\cdot 9\cdot 11}+\frac{1}{9\cdot 11\cdot 13\cdot 15}+\&c.\ldots=\frac{1}{12}S-\frac{1}{18}$;

$m=19, n=16$. $\frac{19}{1\cdot 3\cdot 5\cdot 7}+\frac{35}{5\cdot 7\cdot 9\cdot 11}+\frac{51}{9\cdot 11\cdot 13\cdot 15}+\&c.\ldots=\frac{1}{4}S$.

Cor. If $n:m$ as $4r:3r+2$, the sum of the series can be accurately had; let therefore $n=4r$ and $m=3r+2$, and we shall have

$3r+2$

$$\frac{3r+2}{1\cdot r+1\cdot 2r+1\cdot 3r+1}+\frac{7r+2}{2r+1\cdot 3r+1\cdot 4r+1\cdot 5r+1}+\&c.\ldots=\frac{1}{2r\cdot r+1}.$$

If $r=1$, $\frac{5}{1\cdot 2\cdot 3\cdot 4}+\frac{9}{3\cdot 4\cdot 5\cdot 6}+\frac{13}{5\cdot 6\cdot 7\cdot 9}+\&c.\ldots=\frac{1}{4}$ *;

$r=2$, $\frac{1}{1\cdot 3\cdot 5\cdot 7}+\frac{2}{5\cdot 7\cdot 9\cdot 11}+\frac{3}{9\cdot 11\cdot 13\ 15}+\&c.\ldots=\frac{1}{96}$;

$r=6$, $\frac{5}{1\cdot 7\cdot 13\cdot 19}+\frac{11}{13\cdot 19\cdot 25\cdot 31}+\frac{17}{25\cdot 31\cdot 37\cdot 43}+\&c.\ldots=\frac{1}{336}$.

PROP. IV.

To find the ſum of the infinite ſeries $\frac{m}{r+1\cdot 2r+1\cdot 3r+1\cdot 4r+1}+$
$\frac{m+n}{3r+1\cdot 4r+1\cdot 5r+1\cdot 6r+1}+\frac{m+2n}{5r+1\cdot 6r+1\cdot 7r+1\cdot 8r+1}+\&c.$

This ſeries reſolves itſelf into

$$\frac{a}{r+1\cdot 2r+1\cdot 3r+1}-\frac{a+b}{2r+1\cdot 3r+1\cdot 4r+1}+\frac{a+2b}{3r+1\cdot 4r+1\cdot 5r+1}-\&c.$$

for by reduction it becomes $\frac{3ra-\overline{r+1}\cdot b}{r+1\cdot 2r+1\cdot 3r+1\cdot 4r+1}+$

$$\frac{3ra+3r-1\cdot b}{3r+1\cdot 4r+1\cdot 5r+1\cdot 6r+1}+\frac{3ra+7r-1\cdot b}{5r+1\cdot 6r+1\cdot 7r+1\cdot 8r+1}+\&c.$$ where the denominators are the ſame as in the given ſeries, and the numerators alſo in arithmetic progreſſion; put therefore $3ra-\overline{r+1}\cdot b=m$, $3ra+\overline{3r-1}\cdot b=m+n$, hence $b=\frac{n}{4r}$, $a=\frac{4rm+\overline{r+1}\cdot n}{12r^2}$, which, ſubſtituted in cor. 2. prop. 2. give

$$\frac{m}{r+1\cdot 2r+1\cdot 3r+1\cdot 4r+1}+\frac{m+n}{3r+1\cdot 4r+1\cdot 5r+1\cdot 6r+1}+\&c.\ldots=$$

$$\frac{5r+2\cdot n-4rm}{6r^3}\times S+\frac{2rm-\overline{4r+1}\cdot n}{6r^3}+\frac{2rm-\overline{r-2}\cdot n}{12\cdot r^3\cdot r+1}+\frac{4rm-\overline{2r-1}\cdot n}{12r^2\cdot r+1\cdot 2r+1}.$$

* Vide DE MOIVRE's Miſ. Anal. pag. 134.

Cor.

Cor. 1. In prop. 3. write a for m and $2b$ for n, and we have

$$\frac{a}{1\cdot r+1\cdot 2r+1\cdot 3r+1}+\frac{a+2b}{2r+1\cdot 3r+1\cdot 4r+1\cdot 5r+1}+\&c.\ldots=\frac{2ra-3r+2\cdot b}{3r^4}\times S+\frac{3r+1\cdot b-ra}{3r^4}-\frac{ra+2b}{6r^3\cdot r+1}.$$

Also in this prop. write $a+b$ for m, and $2b$ for n, and we have

$$\frac{a+b}{r+1\cdot 2r+1\cdot 3r+1\cdot 4r+1}+\frac{a+3b}{3r+1\cdot 4r+1\cdot 5r+1\cdot 6r+1}+\&c.\ldots=\frac{3r+2\cdot b-2ra}{3r^4}\times S+\frac{ra-3r+1\cdot b}{3r^4}+\frac{ra+2b}{6r^3\cdot r+1}+\frac{2ra+b}{6r^2\cdot r+1\cdot 2r+1}.$$

subtract this latter series from the former, and we have

$$\frac{a}{1\cdot r+1\cdot 2r+1\cdot 3r+1}-\frac{a+b}{r+1\cdot 2r+1\cdot 3r+1\cdot 4r+1}+\&c.\ldots=\frac{4ra-3r+2\cdot 2b}{3r^4}\times S+\frac{3r+1\cdot 2b-2ra}{3r^4}-\frac{ra+2b}{3r^3\cdot r+1}-\frac{2ra+b}{6r^2\cdot r+1\cdot 2r+1}.$$

Let $r=1$, and we have

$$\frac{a}{1\cdot 2\cdot 3\cdot 4}-\frac{a+b}{2\cdot 3\cdot 4\cdot 5}+\frac{a+2b}{3\cdot 4\cdot 5\cdot 6}-\&c.\ldots=\frac{4a-10b}{3}\times S+\frac{83b-32a}{36}.$$

If $a=1$, $b=0$, $\frac{1}{1\cdot 2\cdot 3\cdot 4}-\frac{1}{2\cdot 3\cdot 4\cdot 5}+\frac{1}{3\cdot 4\cdot 5\cdot 6}-\&c.\ldots=\frac{4}{3}S-\frac{8}{9}$.

$a=3$, $b=1$, $\frac{1}{1\cdot 2\cdot 4}-\frac{1}{2\cdot 3\cdot 5}+\frac{1}{3\cdot 4\cdot 6}-\&c.\ldots=\frac{2}{3}S-\frac{13}{36}$.

Let $r=3$, and we have

$$\frac{a}{1\cdot 4\cdot 7\cdot 10}-\frac{a+b}{4\cdot 7\cdot 10\cdot 13}+\frac{a+2b}{7\cdot 10\cdot 13\cdot 16}-\&c.\ldots=\frac{12a-22b}{243}\times S+\frac{1027b-156a}{3\cdot 81\cdot 56}$$

If $a=1$, $b=1$, $\frac{1}{1\cdot 4\cdot 7\cdot 10}-\frac{2}{4\cdot 7\cdot 10\cdot 13}+\frac{3}{7\cdot 10\cdot 13\cdot 16}-\&c.\ldots=\frac{871}{3\cdot 81\cdot 56}-\frac{10}{3\cdot 81}S$.

$a=4$, $b=3$, $\frac{1}{1\cdot 7\cdot 10}-\frac{1}{4\cdot 10\cdot 13}+\frac{1}{7\cdot 13\cdot 16}-\&c.\ldots=\frac{13}{72}-\frac{2}{27}S$.

Cor. 2. If $a:b$ as $3r+2:2r$, the sum of the series can be accurately found; take $\therefore a=3r+2$, and $b=2r$, and we shall have

$$\frac{3r+2}{1\cdot r+1\cdot 2r+1\cdot 3r+1}-\frac{5r+2}{r+1\cdot 2r+1\cdot 3r+1\cdot 4r+1}+\&c.\ldots=\frac{1}{r+1\cdot 2r+1}.$$

If

If $r=1$, $\frac{5}{1.2.3.4}-\frac{7}{2.3.4.5}+\frac{9}{3.4.5.6}-\&c.\ldots=\frac{1}{60}$;

$r=2$, $\frac{2}{1.3.5.7}-\frac{3}{3.5.7.9}+\frac{4}{5.7.9.11}-\&c.\ldots=\frac{1}{15}$;

$r=3$, $\frac{11}{1.4.7.10}-\frac{17}{4.7.10.13}+\frac{23}{7.10.13.16}-\&c.\ldots=\frac{1}{28}$.

PROP. V.

To find the ſum of the infinite ſeries $\frac{m}{1.r+1.2r+1.3r+1.4r+1}+$
$\frac{m+n}{2r+1.3r+1.4r+1.5r+1.6r+1}+\frac{m+2n}{4r+1.5r+1.6r+1.7r+1.8r+1}+\&c.$

This ſeries reſolves itſelf into $\frac{a}{1.r+1.2r+1.3r+1}-$
$\frac{a+b}{r+1.2r+1.3r+1.4r+1}+\frac{a+2b}{2r+1.3r+1.4r+1.5r+1}-\&c.$; for by reduction this ſeries becomes $\frac{4ra-b}{1.r+1.2r+1.3r+1.4r+1}+$
$\frac{4ra+\overline{6r-1}.b}{2r+1.3r+1.4r+1.5r+1.6r+1}+\frac{4ra+\overline{12r-1}.b}{4r+1.5r+1.6r+1.7r+1.8r+1}+\&c.$
where the numerators are in arithmetic progreſſion, and the denominators the ſame as in the given ſeries; aſſume therefore $4ra-b=m$, $4ra+\overline{6r-1}.b=m+n$, hence $b=\frac{n}{6r}$, $a=\frac{6rm+n}{24r^2}$, which values being ſubſtituted in cor. 1. prop. 4. give

$\frac{m}{1.r+1.2r+1.3r+1.4r+1}+\frac{m+n}{2r+1.3r+1.4r+1.5r+1.6r+1}+\&c.\ldots=$
$\frac{2rm-2r+1.n}{6r^3}\times S+\frac{4r+1.n-2rm}{12r^5}-\frac{6rm+9n}{72.r^4.r+1}-\frac{2rm+n}{24r^3.r+1.2r+1}$.

Let $r=1$, and we have

$\frac{m}{1.2.3.4.5}+\frac{m+n}{3.4.5.6.7}+\frac{m+2n}{5.6.7.8.9}+\&c.\ldots=\frac{2m-3n}{6}\times S+\frac{25n-16m}{72}$.

If

If $m=1, n=0$, $\frac{1}{1.2.3.4.5}+\frac{1}{3.4.5.6.7}+\frac{1}{5.6.7.8.9}+\&c.\ldots=\frac{1}{3}S-\frac{2}{9}$;

$m=1, n=1$, $\frac{1}{1.2.3.4.5}+\frac{2}{3.4.5.6.7}+\frac{3}{5.6.7.8.9}+\&c.\ldots=\frac{1}{8}-\frac{1}{6}S$;

$m=4, n=2$, $\frac{1}{1.2.3.5}+\frac{1}{3.4.5.7}+\frac{1}{5.6.7.9}+\&c.\ldots=\frac{1}{3}S-\frac{7}{36}$.

$m=25, n=16$, $\frac{25}{1.2.3.4.5}+\frac{41}{3.4.5.6.7}+\frac{57}{5.6.7.8.9}+\&c.\ldots=\frac{1}{3}S$.

Let $r=2$, and we have $\frac{m}{1.3.5.7.9}+\frac{m+n}{5.7.9.11.13}+\frac{m+2n}{9.11.13.15.17}+\&c.\ldots=\frac{4m-5n}{192}\times S+\frac{59n-44m}{24.120}$.

If $m=1, n=0$, $\frac{1}{1.3.5.7.9}+\frac{1}{5.7.9.11.13}+\frac{1}{9.11.13.15.17}+\&c.\ldots=\frac{1}{48}S-\frac{11}{24.30}$;

$m=3, n=2$, $\frac{1}{1.5.7.9}+\frac{1}{5.9.11.13}+\frac{1}{9.13.15.17}+\&c.\ldots=\frac{1}{96}S-\frac{7}{24.60}$.

Cor. If $n:m$ as $2r:2r+1$, the ſum of the ſeries can be accurately found; aſſume therefore $n=2r$, $m=2r+1$, and we have

$\frac{1}{1.r+1.3r+1.4r+1}+\frac{1}{2r+1.3r+1.5r+1.6r+1}+\&c.\ldots=\frac{1}{6.r.r+1.2r+1}$.

If $r=1$, $\frac{1}{1.2.4.5}+\frac{1}{3.4.6.7}+\frac{1}{5.6.8.9}+\&c.\ldots=\frac{1}{36}$;

$r=3$, $\frac{1}{1.4.10.13}+\frac{1}{7.10.16.19}+\frac{1}{13.16.22.25}+\&c.\ldots\frac{1}{504}$.

Having thus far explained the method of ſummation of ſuch ſeries as I propoſed to treat of in the firſt part of this paper, I truſt it is not neceſſary to ſay any thing further, as the ſame method of proceeding will manifeſtly continue the ſeries to any propoſed number of factors in the denominator; I ſhall therefore conclude with pointing out a remarkable property of thoſe ſeries whoſe ſum can be accurately found: that when the number of factors in the denominator is *even*, the numerator is always equal to the ſum of the two

middle factors; and when the number of factors be *odd*, the numerator will be equal to the middle factor, and consequently will take it out of the denominator, and leave a series whose numerators are unity, and whose denominators want the middle factor.

The method of summation of series here made use of may also be applied in investigating the sums of a great variety of other series; but as a further application of this method would carry us beyond the limits to which this paper must be confined, I shall re-assume the subject at some future opportunity, and proceed immediately to the second part.

PART II.

PROP.

To find the sum of the infinite series $\frac{p}{n \cdot n+m \ldots . n+rm} +$ $\frac{q}{n+m \ldots . n+r+1 \cdot m} + \frac{s}{n+2m \ldots . n+r+2 \cdot m}$ + &c. *when the last differences of the numerators become equal to nothing.*

Assume $a+nb+n \cdot \overline{n+m} \cdot c+n \cdot \overline{n+m} \cdot \overline{n+2m} \cdot d+$ &c. to any number (r) of terms; then, if for n we write $n+m$, $n+2m$, $n+3m$, &c. successively, there will result a series of quantities

whose rth difference is $=0$; ſubſtitute, therefore, this ſeries of quantities for p, q, s, &c. reſpectively, and the given ſeries becomes

$$\frac{a+nb+n\cdot n+m\cdot c+\&c.}{n\cdot n+m\ldots\ldots n+rm}+\frac{a+n+m\cdot b+n+m\cdot n+2m\cdot c+\&c.}{n+m\ldots\ldots n+r+1\cdot m}+$$

$$\frac{a+n+2m\cdot b+n+2m\cdot n+3m\cdot c+\&c.}{n+2m\ldots\ldots n+r+2\cdot m}+\&c.$$

which manifeſtly reſolves itſelf into the following ſeries

$$\frac{a}{n\cdot n+m\ldots\ldots n+rm}+\frac{a}{n+m\ldots\ldots n+r+1\cdot m}+\frac{a}{n+2m\ldots\ldots n+r+2\cdot m}+\&c.$$

$$\frac{b}{n+m\ldots\ldots n+rm}+\frac{b}{n+2m\ldots\ldots n+r+1\cdot m}+\frac{b}{n+3m\ldots\ldots n+r+2\cdot m}+\&c.$$

$$\frac{c}{n+2m\ldots\ldots n+rm}+\frac{c}{n+3m\ldots\ldots n+r+1\cdot m}+\frac{c}{n+4m\ldots\ldots n+r+2\cdot m}+\&c.$$

&c. &c. &c.

where the number of ſeries is r, the ſum of each of which being taken by a well known rule, the ſum of the given ſeries becomes

$$\frac{a}{n\cdot n+m\ldots\ldots n+r-1\cdot m\cdot m\cdot r}+\frac{b}{n+m\ldots\ldots n+r-1\cdot m\cdot m\cdot r-1}+$$

$$\frac{c}{n+2m\ldots\ldots n+r-1\cdot m\cdot m\cdot r-2}+\&c.$$

where the law of continuation is manifeſt.

CASE 1. *To find the ſum of the infinite ſeries* $\frac{3}{1\cdot2\cdot3\cdot4}+\frac{6}{2\cdot3\cdot4\cdot5}+\frac{10}{3\cdot4\cdot5\cdot6}+\frac{15}{4\cdot5\cdot6\cdot7}+\&c.$

Here $n=1$, $m=1$, $r=3$, and the third differences become $=0$; therefore $a+b+2c=3$, $a+2b+6c=6$, $a+3b+12c=10$, conſequently $a=1$, $b=1$, $c=\frac{1}{2}$, and therefore the ſum ſought will be $\frac{1}{1\cdot2\cdot3\cdot3}+\frac{1}{2\cdot3\cdot2}+\frac{1}{2\cdot3}=\frac{11}{36}$.

CASE

CASE 2. *To find the ſum of the infinite ſeries* $\frac{1.2}{1.3.5.7} + \frac{2.3}{3.5.7.9} + \frac{3.4}{5.7.9.11}$ &c.

In this caſe $n=1$, $m=2$, $r=3$, and the 3d differences become $=0$; therefore $a+b+3c=2$, $a+3b+15c=6$, $a+5b+35c=12$, conſequently $a=\frac{1}{4}$, $b=\frac{1}{4}$, $c=\frac{1}{4}$, and hence the ſum of the required ſeries becomes $\frac{3}{1.3.5.2.3.4} + \frac{1}{3.5.2.2.2} + \frac{1}{5.2.4} = \frac{1}{24}$.

CASE 3. *To find the ſum of the infinite ſeries* $\frac{1}{3.4.5.6.7.8} + \frac{2}{4.5.6.7.8.9} + \frac{4}{5.6.7.8.9.10} + \frac{12}{6.7.8.9.10.11} + \frac{31}{7.8.9.10.11.12}$ &c.

Here $n=3$, $m=1$, $r=5$, and the 4th differences become $=0$; therefore $a+3b+12c+60d=1$, $a+4b+20c+120d=2$, $a+5b+30c+210d=4$, $a+6b+42c+336d=12$, conſequently $a=-54$, $b=47$, $c=-12$, $d=\frac{5}{6}$, therefore the ſum of the given ſeries is $\frac{-46}{3.4.5.6.7.5} + \frac{47}{4.5.6.7.4} - \frac{12}{5.6.7.3} + \frac{5}{6.7.2.6} = \frac{61}{50400}$.

CASE 4. *To find the ſum of the infinite ſeries* $\frac{1}{1.4.7.10} + \frac{9}{4.7.10.13} + \frac{25}{7.10.13.16}$ &c.

Here $n=1$, $m=3$, $r=3$, and the 3d differences are $=0$; therefore $a+b+4c=1$, $a+4b+28c=9$, $a+7b+70c=25$, conſequently $a=\frac{1}{9}$, $b=\frac{-8}{9}$, $c=\frac{4}{9}$; $\therefore$ the ſum of the given ſeries will be $\frac{1}{1.4.7.3.3.9} - \frac{8}{4.7.3.2.9} + \frac{4}{7.3.9} = \frac{37}{2208}$.

CASE 5. *To find the Sum of the infinite Series* $\frac{1}{1\cdot3\cdot5\cdot7\cdot9} + \frac{4}{3\cdot5\cdot7\cdot9\cdot11} + \frac{10}{5\cdot7\cdot9\cdot11\cdot13} + \frac{20}{7\cdot9\cdot11\cdot13\cdot15}$.

In this caſe $n=1$, $m=2$, $r=4$, and the 4th differences become $=0$; therefore $a+b+3c+15d=1$, $a+3b+15c+105d=4$, $a+5b+35c+315d=10$, $a+7b+63c+693d=20$, conſequently $a=\frac{5}{16}$, $b=\frac{3}{16}$, $c=\frac{1}{16}$, $d=\frac{1}{48}$, and hence the ſum of the given ſeries becomes $\frac{5}{1\cdot3\cdot5\cdot7\cdot2\cdot4\cdot16} + \frac{3}{3\cdot5\cdot7\cdot2\cdot3\cdot16} + \frac{1}{5\cdot7\cdot2\cdot2\cdot16} + \frac{1}{7\cdot2\cdot48} = \frac{1}{384}$.

This propoſition may alſo be applied to find the ſum of all thoſe ſeries whoſe numerators being unity, the denominators ſhall be deficient by any number of correſponding terms, however taken: for as the product of all ſuch factors muſt form a progreſſion, whoſe differences will become equal to nothing, if ſuch products be aſſumed for the numerators of the given ſeries having its factors compleated, another ſeries will be formed equal to the given ſeries, whoſe ſum can be found by this propoſition.

CASE 1. *To find the ſum of the infinite ſeries* $\frac{1}{1\cdot2\cdot4\cdot6} + \frac{1}{2\cdot3\cdot5\cdot7} + \frac{1}{3\cdot4\cdot6\cdot8}$ + &c.

By completing the factors in the denominators, and multiplying the numerators by the ſame quantities the given ſeries becomes $\frac{15}{1\cdot2\cdot3\cdot4\cdot5\cdot6} + \frac{24}{2\cdot3\cdot4\cdot5\cdot6\cdot7} + \frac{35}{3\cdot4\cdot5\cdot6\cdot7\cdot8}$ + &c. in which caſe $n=1$, $m=1$, $r=5$, and the 3d differences become $=0$;

$=0$; therefore $a+b+2c=15$, $a+2b+6c=24$, $a+3b+12c=35$, conſequently $a=8$, $b=5$, $c=1$, and therefore the ſum of the ſeries required is $\frac{8}{1\cdot2\cdot3\cdot4\cdot5\cdot5}+\frac{5}{2\cdot3\cdot4\cdot5\cdot4}+\frac{1}{3\cdot4\cdot5\cdot3}=\frac{211}{7200}$.

CASE 2. *To find the ſum of the infinite ſeries* $\frac{1}{1\cdot5\cdot11}+\frac{1}{3\cdot7\cdot13}+\frac{1}{5\cdot9\cdot15}+$&c.

This ſeries, when completed, becomes $\frac{189}{1\cdot3\cdot5\cdot7\cdot9\cdot11}+\frac{495}{3\cdot5\cdot7\cdot9\cdot11\cdot13}+\frac{1001}{5\cdot7\cdot9\cdot11\cdot13\cdot15}+\frac{1755}{7\cdot9\cdot11\cdot13\cdot15\cdot17}+$ &c. where $n=1$, $m=2$, $r=5$, and the 4th differences are $=0$; therefore $a+b+3c+15d=189$, $a+3b+15c+105d=495$, $a+5b+35c+315d=1001$, $a+7b+63c+693d=1755$, conſequently $a=96$, $b=48$, $c=10$, $d=1$; and hence the ſum of the given ſeries is $\frac{96}{1\cdot3\cdot5\cdot7\cdot9\cdot2\cdot5}+\frac{48}{3\cdot5\cdot7\cdot9\cdot2\cdot4}+\frac{10}{5\cdot7\cdot9\cdot2\cdot3}+\frac{1}{7\cdot9\cdot2\cdot2}=\frac{487}{18900}$.

CASE 3. *To find the ſum of the infinite ſeries* $\frac{1}{1\cdot4\cdot13\cdot16}+\frac{1}{4\cdot7\cdot16\cdot19}+\frac{1}{7\cdot10\cdot19\cdot22}+$&c.

This ſeries reſolves itſelf into $\frac{70}{1\cdot4\cdot7\cdot10\cdot13\cdot16}+\frac{130}{4\cdot7\cdot10\cdot13\cdot16\cdot19}+\frac{208}{7\cdot10\cdot13\cdot16\cdot19\cdot22}+$ &c. where $n=1$, $m=3$, $r=5$, and the 3d differences $=0$; therefore $a+b+4c=70$, $a+4b+28c=130$, $a+7b+70c=208$, from whence $a=54$, $b=12$, $c=1$; therefore the ſum of the given ſeries is

$\frac{54}{1\cdot4\cdot7\cdot10\cdot13\cdot3\cdot5}+\frac{12}{4\cdot7\cdot10\cdot13\cdot3\cdot4}+\frac{1}{7\cdot10\cdot13\cdot3\cdot3}=\frac{227}{03800}$.

By this propoſition we may alſo inveſtigate the ſum of the ſeries when there are any number of deficient terms in the denominators, and where the laſt differences of the numerators become equal to nothing; for if the factors in the denominators be completed, and the numerators be multiplied by the ſame quantities, their differences will ſtill become equal to nothing.

CASE 1. *To find the ſum of the infinite ſeries* $\frac{1}{1\cdot3\cdot4\cdot6}+\frac{3}{2\cdot4\cdot5\cdot7}+\frac{6}{3\cdot5\cdot6\cdot8}+\frac{10}{4\cdot6\cdot7\cdot9}+\frac{15}{5\cdot7\cdot8\cdot10}+$&c.

This ſeries, by completing the factors in the denominators and multiplying the numerators by the ſame quantities, becomes $\frac{10}{1\cdot2\cdot3\cdot4\cdot5\cdot6}+\frac{54}{2\cdot3\cdot4\cdot5\cdot6\cdot7}+\frac{168}{3\cdot4\cdot5\cdot6\cdot7\cdot8}+$&c. in which caſe $n=1$, $m=1$, $r=5$, and as the 5th differences are $=0$:

$\therefore a+b+2c+6d+24e=10$, $a+2b+6c+24d+120e=54$, $a+3b+12c+60d+360e=168$, $a+4b+20c+120d+840e=400$, $a+5b+30c+210d+1680e=810$, from whence $a=0$, $b=0$, $c=-1$, $d=0$, $e=\frac{1}{2}$, conſequently the ſum of the given ſeries is $=-\frac{1}{3\cdot4\cdot5\cdot3}+\frac{1}{5\cdot2}=\frac{17}{180}$.

CASE 2. *To find the ſum of the infinite ſeries* $\frac{1}{1\cdot3\cdot7\cdot9}+\frac{5}{3\cdot5\cdot9\cdot11}+\frac{11}{5\cdot7\cdot11\cdot13}+\frac{19}{7\cdot9\cdot13\cdot15}+$&c.

By proceeding as before this ſeries becomes $\frac{6}{1\cdot3\cdot5\cdot7\cdot9}+$

$\frac{35}{3\cdot5\cdot7\cdot9\cdot11}+\frac{99}{5\cdot7\cdot9\cdot11\cdot13}+$&c. where $n=1$, $m=2$, $r=4$, and the 4th differences $=0$; therefore $a+b+3c+15d=5$, $a+3b+15c+105d=35$, $a+5b+35c+315d=99$, $a+7b+63c+693d=209$, conſequently $a=-1$, $b=\frac{1}{2}$, $c=\frac{1}{8}$, $d=\frac{1}{4}$, therefore the ſum of the given ſeries $=\frac{-1}{1\cdot3\cdot5\cdot7\cdot2\cdot4}+\frac{3}{3\cdot5\cdot7,2\cdot3\cdot4}+\frac{1}{5\cdot7\cdot2\cdot2\cdot2}+\frac{1}{7\cdot2\cdot4}=\frac{3}{140}$.

By a method ſimilar to that made uſe of in this propoſition may any number of factors be taken from the denominators of thoſe ſeries delivered in part the firſt, and alſo from a great variety of others; but as the examples here given muſt be ſufficient to point out the method of proceeding in all other caſes, we will proceed to the third part.

PART III.

THE ſum of every converging infinite ſeries, whoſe terms ultimately become equal to nothing, may always be exhibited by the ſum of another ſeries formed by collecting two or more terms of the former ſeries into one. This is not true, however, where the terms of the infinite ſeries continally diverge, or converge to any aſſignable quantity, and

and are affected with the signs +, −, alternately: for inſtance, the ſeries $\frac{1}{1} - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} +$ &c. if we collect two terms into one, beginning at the firſt term, will become $-\frac{1}{2 \cdot 3} - \frac{1}{4 \cdot 5} - \frac{1}{6 \cdot 7} +$ &c. If we begin at the ſecond term it becomes $\frac{1}{1 \cdot 2} + \frac{1}{3 \cdot 4} + \frac{1}{5 \cdot 6} +$ &c.; neither of which gives the ſum of the aſſumed ſeries; but in this, and every other caſe of the like nature, a correction will be neceſſary: to determine the value of which, and from whence the neceſſity thereof ariſes, is the ſubject of this third part.

LEMMA.

If r be any quantity whatever, then will $\frac{1}{2r} = \frac{1}{r} - \frac{1}{r} + \frac{1}{r} - \frac{1}{r} +$ &c. *ad infinitum.*

For $\frac{1}{2r} = \frac{1}{r+r} =$ (by common diviſion) $\frac{1}{r} - \frac{1}{r} + \frac{1}{r} - \frac{1}{r} +$ &c. ad infinitum.

Cor. 1. Hence $-\frac{1}{2r} = -\frac{1}{r} + \frac{1}{r} - \frac{1}{r} + \frac{1}{r} -$ &c. ad infinitum.

Cor. 2. Hence alſo $\frac{x}{2v} = \frac{x}{v} - \frac{x}{v} + \frac{x}{v} - \frac{x}{v} +$ &c. ad infinitum;

and $-\frac{x}{2v} = -\frac{x}{v} + \frac{x}{v} - \frac{x}{v} + \frac{x}{v} -$ &c. ad infinitum.

PROP. I.

If $\frac{rn+m}{n}$ *be the general term of a ſeries formed by writing for n any ſeries of numbers in arithmetic progreſſion, and whoſe ſigns are alternately + and −; then if a ſeries be formed by col-*

lecting two terms into one, beginning at the firſt term, the ſum of the ſeries thence ariſing will be leſs than the ſum of the given ſeries by $\frac{1}{2r}$. *If a ſeries be formed by beginning at the ſecond Term, the ſum thereof will be greater than the ſum of the given ſeries by* $\frac{1}{2r}$.

For let $\frac{n}{rn+m} - \frac{n+a}{n+a.r+m}$ be any two ſucceſſive terms of the ſeries, which, if we begin to collect at the firſt term (the firſt term being +) will be two terms to be collected into one, and which will therefore give $\frac{-am}{rn+m \times n+a.r+m}$ for a general term of the reſulting ſeries. Let us now make n infinite, and then the denominator of this term becomes infinite, and the numerator finite; therefore the terms of this latter ſeries at an infinite diſtance becoming infinitely ſmall, the ſeries will there terminate. Now, by making n infinite in the given ſeries, the two ſucceſſive general terms at an infinite diſtance become $\frac{1}{r} - \frac{1}{r}$; conſequently *this* ſeries is ſtill continued after the *other* terminates; and the terms of ſuch a continuation will be (as they begin with $\frac{n}{rn+m} - \frac{n+a}{n+a.r+m}$ by making n infinite) $\frac{1}{r} - \frac{1}{r} + \frac{1}{r} - \frac{1}{r} +$ &c. which will alſo be continued ad infin. and whoſe ſum by the lemma is $\frac{1}{2r}$; conſequently the given ſeries exceeds that which is formed by collecting two terms into one, beginning at the firſt, by $\frac{1}{2r}$; hence the ſum of the *latter* ſeries $+\frac{1}{2r}$ will be equal to the ſum of the *former*. If we begin to

collect at the second term, then will $-\frac{n}{rn+m}+\frac{n+a}{n+a.r+m}$ be the two successive general terms of the given series to be collected into one; consequently the continuation of the given series when n becomes infinite will be $-\frac{1}{r}+\frac{1}{r}-\frac{1}{r}+\frac{1}{r}-$ &c. ad infinitum, whose sum, by cor. 1. to the lem. is $-\frac{1}{2r}$; in this case, therefore, the sum of the given series is less than the sum of the series formed by collecting two terms into one, beginning at the second term, by $\frac{1}{2r}$; hence the sum of the *latter* series $-\frac{1}{2r}$ will be equal to the sum of the *former*.

CASE 1. *Let the given series be* $\frac{1}{2}-\frac{2}{3}+\frac{3}{4}-\frac{4}{5}+$&c.

Here $r=1$, $n=1, 2, 3, 4,$ &c. and $m=1$. Now, if we begin to collect at the first term, the series resolves itself into $-\frac{1}{2\cdot 3}-\frac{1}{4\cdot 5}-\frac{1}{6\cdot 7}-$ &c. and the correction, to be *added*, being $\frac{1}{2}$, we have $-\frac{1}{2\cdot 3}-\frac{1}{4\cdot 5}-\frac{1}{6\cdot 7}-$&c. $+\frac{1}{2}$ for the sum of the given series. Now $-\frac{1}{2\cdot 3}-\frac{1}{4\cdot 5}-\frac{1}{6\cdot 7}-$ &c. is well known to be equal to $-1+$ hyp. log. of 2; consequently the sum of the given series is $=-\frac{1}{2}+$ hyp. log. of 2.

If we begin to collect at the second term, the series becomes $\frac{1}{1\cdot 2}+\frac{1}{3\cdot 4}+\frac{1}{5\cdot 6}+$&c. and the correction, to be *subtracted*, being $\frac{1}{2}$, we have $\frac{1}{1\cdot 2}+\frac{1}{3\cdot 4}+\frac{1}{5\cdot 6}+$&c. $-\frac{1}{2}$ for the sum of the given series;

ſeries; but $\frac{1}{1\cdot 2}+\frac{1}{3\cdot 4}+\frac{1}{5\cdot 6}+$ &c. is equal to the hyp. log. of 2; therefore the ſum of the given ſeries is $=-\frac{1}{2}+$ hyp. log. of 2, the ſame as before.

CASE 2. *Let the given ſeries be* $\frac{1}{3}-\frac{2}{5}+\frac{3}{7}+\frac{4}{9}+$ &c.

Here $r=2$, $n=1$, 2, 3, 4, &c. $m=1$. Now, if we begin to collect at the ſecond term, the ſeries becomes $\frac{1}{1\cdot 3}+\frac{1}{5\cdot 7}+\frac{1}{9\cdot 11}+$ &c. and the correction, to be *ſubtracted*, being $\frac{1}{4}$, we have $\frac{1}{1\cdot 3}+\frac{1}{5\cdot 7}+\frac{1}{9\cdot 11}+$ &c. $-\frac{1}{4}$ for the ſum of the given ſeries; but $\frac{1}{1\cdot 3}+\frac{1}{5\cdot 7}+\frac{1}{9\cdot 11}+$ &c. is equal to a circular arc (A) of $22^{\circ}\frac{1}{2}$, whoſe radius is unity; therefore the ſum of the given ſeries $=A-\frac{1}{4}$.

PROP. II.

If $\frac{x+nz}{w+nv}$ *be the general term of a ſeries formed by writing for* n *any ſeries of numbers in arithmetic progreſſion, and whoſe terms are alternately* + *and* −; *then if a ſeries be formed by collecting two terms into one, beginning at the firſt term, the ſum of the ſeries thence ariſing will be leſs than the ſum of the given ſeries by* $\frac{z}{2v}$. *If a ſeries be formed by beginning at the ſecond term, the ſum thereof will be greater than the ſum of the given ſeries by* $\frac{z}{2v}$.

For let $\frac{x+nz}{w+nv} - \frac{x+\overline{n+a}\cdot z}{w+\overline{n+a}\cdot v}$ be any two ſucceſſive termsof the given ſeries, which, if we begin to collect at the firſt term, will be the two general terms to be collected into one, and will therefore give $\frac{axv-awz}{w+nv \times w+n+a\cdot v}$ for a general term of the reſulting ſeries. Let us now make n infinite and thenthis term will vaniſh, and conſequently the reſulting ſeries will terminate at an infinite diſtance. Now, by making n infinite in the given ſeries, the two ſucceſſive terms (as they begin with $\frac{x+nz}{w+nv} - \frac{x+\overline{n+a}\cdot z}{w+n+a\cdot v}$ by making n infinite) become $\frac{z}{v} - \frac{z}{v}$; this ſeries, therefore, is ſtill continued after the other terminates; and the terms of ſuch a continuation will be $\frac{z}{v} - \frac{z}{v} + \frac{z}{v} - \frac{z}{v}$ +&c. ad infinitum, and whoſe ſum by cor. 2. to the lem. is $\frac{z}{2v}$; conſequently the given ſeries exceeds that which is formed by collecting two terms into one, beginning at the firſt, by $\frac{z}{2v}$; hence the ſum of the *latter* ſeries $+\frac{z}{2v}$ will be equal to the ſum of the *former*. Now, if we begin to collect at the ſecond term, then will $-\frac{x+nz}{w+nv} + \frac{x+\overline{n+a}\cdot z}{w+\overline{n+a}\cdot v}$ be two general terms of the given ſeries to be collected into one; conſequently the continuation of the given ſeries, when n becomes infinite, will be $-\frac{z}{v} + \frac{z}{v} - \frac{z}{v} + \frac{z}{v}$ -&c. ad infinitum, whoſe ſum by cor. 2. to the lem. is $-\frac{z}{2v}$; in this caſe, therefore, the ſum of the given ſeries is leſs than the ſum of the ſeries formed by collecting two terms into one, beginning at the ſecond term, by

$\frac{z}{2v}$; hence the ſum of the *latter* ſeries $-\frac{z}{2v}$ will be equal to the ſum of the *former*.

CASE 1. *Let the given ſeries be* $\frac{7}{3}-\frac{11}{5}+\frac{15}{7}-\frac{19}{9}+$&c.

Here $x=3$, $z=2$, $w=1$, $v=1$, $n=2, 4, 6, 8,$ &c. Now, if we begin to collect at the firſt term, the ſeries becomes $\frac{2}{3\cdot5}+\frac{2}{7\cdot9}+\frac{2}{11\cdot13}+$&c. and the correction, to be *added*, being 1, we have $\frac{2}{3\cdot5}+\frac{2}{7\cdot9}+\frac{2}{11\cdot13}+$&c. $+1$ for the ſum of the given ſeries; but if A = a circular arc of 45° whoſe radius is unity, it is well known that $\frac{2}{3\cdot5}+\frac{2}{7\cdot9}+\frac{2}{11\cdot13}+$ &c. $=1-$A; therefore the ſum of the given ſeries is $2-$A.

CASE 2. *Let the given ſeries be* $\frac{16}{1}-\frac{27}{2}+\frac{38}{3}-\frac{49}{4}+$&c.

Here $w=1$, $v=1$, $x=16$, $z=11$, $n=0, 1, 2, 3,$ &c. Now, if we begin to collect at the firſt term, the ſeries becomes $\frac{5}{1\cdot2}+\frac{5}{3\cdot4}+\frac{5}{5\cdot6}+$&c. and the correction, to be *added*, being $\frac{11}{2}$, we have $\frac{5}{1\cdot2}+\frac{5}{3\cdot4}+\frac{5}{5\cdot6}+$&c. $+\frac{11}{2}$ for the ſum of the given ſeries; but $\frac{5}{1\cdot2}+\frac{5}{3\cdot4}+\frac{5}{6\cdot7}+$&c. is equal to 5 × hyp. log. of 2, conſequently the ſum of the given ſeries is equal to $\frac{11}{2}+5\times$ hyp. log. of 2.

Becauſe $\frac{axv-awx}{w+nv\times w+n+a\cdot v}$, the general term of the ſeries formed

formed by reducing two terms into one, has its numerator independent of the value of, n, it is manifeſt, that the numerators of that ſeries will be all equal. Now, if a ſeries be aſſumed, the numerators of whoſe terms are unity, and in every other reſpect the ſame as the ſeries in this propoſition, that is, if $\frac{1}{w+nv} - \frac{1}{w+n+a.v}$ be two ſucceſſive terms of a ſeries, it is manifeſt, that if every two terms of this ſeries be reduced into one, the general term of the reſulting ſeries will be $\frac{-av}{w+nv \times w+n+a.v}$, where the numerator is a conſtant quantity $-av$; conſequently the ſum of the ſeries whoſe general term is $\frac{avx-awz}{w+nv \times w+n+a.v}$ is to the ſum of the ſeries whoſe general term is $\frac{-av}{w+nv \times w+n+a.v}$ as $vx-wz$ to $-v$, or in a given ratio; whenever, therefore, the ſum of the latter ſeries can be found, the ſum of the former can be found, and conſequently, after proper correction, the ſum of the ſeries in this propoſition can be found.

Hence, therefore, in the two caſes given above in whatever arithmetic progreſſion the numerators may proceed, the ſum of the former can always be expreſſed by circular arcs, and the latter by the hyp. log. of 2.

Hence alſo, as it appears from lem. 1. part the firſt, that the ſum of the ſeries $\frac{1}{1} - \frac{1}{r+1} + \frac{1}{2r+1} - \frac{1}{3r+1} +$ &c. can always be expreſſed by circular arcs and logarithms, it is manifeſt, that if the numerators form any arithmetic progreſſion, the ſum of ſuch ſeries may be found by this propoſition, and will always be exhibited by circular arcs and logarithms.

Beſides

Beſides the ſeries contained in the foregoing propoſitions, a great variety of other ſeries might be produced where a correction is neceſſary, after collecting two terms into one, in order to exhibit the true value of the given ſeries. As the proper correction, however, may always be found from the principles delivered in the above propoſitions, that is, by conſidering what the terms of the given ſeries become at an infinite diſtance, I ſhall only add one or two inſtances more, and conclude what I at preſent intend to offer on this ſubject.

EX. 1. *Let it be required to find the ſum of the infinite ſeries* $\frac{3\cdot4}{1\cdot2}-\frac{4\cdot5}{2\cdot3}+\frac{5\cdot6}{3\cdot4}-\frac{6\cdot7}{4\cdot5}+$&c.

This, by reſolving two terms into one, becomes $\frac{16}{1\cdot2\cdot3}+\frac{24}{3\cdot4\cdot5}+\frac{32}{5\cdot6\cdot7}-$&c.; and as the terms of the given ſeries continually approach to unity, the correction, to be *added*, is $\frac{1}{2}$, conſequently $\frac{16}{1\cdot2\cdot3}+\frac{24}{3\cdot4\cdot5}+\frac{32}{5\cdot6\cdot7}-$&c.$+\frac{1}{2}$ is equal to the ſum of the given ſeries; but by prop. 1. part I. the ſum of the ſeries $\frac{16}{1\cdot2\cdot3}+\frac{24}{3\cdot4\cdot5}+\frac{32}{5\cdot6\cdot7}+$&c. is equal to $8S-2$ (S being the hyp. log. 2.) conſequently the ſum of the given ſeries is $8S-1\frac{1}{2}$.

EX. 2. *Let it be required to find the ſum of the infinite ſeries* $\frac{1\cdot2}{1\cdot3}-\frac{2\cdot3}{3\cdot5}+\frac{3\cdot4}{5\cdot7}-\frac{4\cdot5}{7\cdot9}+$&c.

This ſeries, by reſolving two terms into one, becomes $\frac{4}{1\cdot3\cdot5}+\frac{8}{5\cdot7\cdot9}+\frac{12}{9\cdot11\cdot13}+$&c. and as the terms of the given

ſeries continually approach to $\frac{1}{4}$, the correction, to be *added*, will be $\frac{1}{8}$, therefore $\frac{4}{1\cdot3\cdot5}+\frac{8}{5\cdot7\cdot9}+\frac{12}{9\cdot11\cdot13}+$ &c. $+\frac{1}{8}$ is = to the ſum of the given ſeries; but by prop. 1. part I. the ſum of $\frac{4}{1\cdot3\cdot5}+\frac{8}{5\cdot7\cdot9}+\frac{12}{9\cdot11\cdot13}+$ &c. is equal to $\frac{1}{4}S+\frac{1}{8}$ (S being a circular arc of 45°, whoſe radius is unity) hence the ſum of the given ſeries is $\frac{1}{4}S+\frac{1}{4}$.

This method is not only applicable to thoſe caſes, where the given ſeries reſolves itſelf into another, whoſe ſum is either accurately known or can be expreſſed by circular arcs and logarithms, but alſo to thoſe caſes where we want to approximate to the value of the given ſeries, as it muſt, in general, be neceſſary firſt to render the terms of the ſeries converging, by collecting two into one, before the operation of approximation begins, and conſequently a correction of this latter is neceſſary in order to exhibit the value of the given ſeries.

XXVI. *A new Method of finding the equal Roots of an Equation, by Diviſion. By the Rev.* John Hellins, *Curate of* Conſtantine, *in* Cornwall; *communicated by* Nevil Maſkelyne, *D. D. F. R. S. and Aſtronomer Royal.*

Read June 20, 1782.

THE following theorems are a production of juvenile years. They were invented about twelve years ago, when algebra was my favourite ſtudy; and one of them (the firſt) was publiſhed as a ſpecimen of this method of extracting the equal roots of an equation about ten years ago. Since that time my avocations have left me but very little leiſure for improving any invention of this kind. Theſe theorems, then, are in their crude ſtate; however, ſuch as they are, I flatter myſelf, they will afford an eaſier ſolution of equations that have equal roots than is generally known, and be acceptable to the ingenious algebraiſt.

THEOREM I.

If the cubic equation $x^3 - px^2 + qx - r = 0$ *has two equal roots, each of them will be* $(x) = \frac{pq - 9r}{2pp - 6q}$.

DEMONSTRATION.

Call the three roots a, a, and b; then, by the compoſition of equations we ſhall have $x^3 \begin{matrix} -2a \\ -b \end{matrix} \Big\} x^2 \begin{matrix} +aa \\ +2ab \end{matrix} \Big\} x - aab = 0$, where

$2a+b=p$, $aa+2ab=q$, and $aab=r$; which values being written in our theorem, we have $x\left(=\frac{pq-9r}{2pp-6q}\right)=$ $\frac{2aaa+4aab+aab+2abb-9aab}{8aa+8ab+2bb-6aa-12ab}=\frac{2aaa-4aab+2abb}{2aa-4ab+2bb}=a$. Q. E. D.

EXAMPLE I.

If the equation $x^3+5x^2-32x+36=0$ has two equal roots, it is propoſed to find them by the above theorem.

Here $p=-5$, $q=-32$, and $r=-36$; theſe values being written in the theorem, we have $\frac{-5\times-32-9\times-36}{2\times25-6\times-32}=\frac{160+324}{50+192}$ $=\frac{484}{242}=2$, which being written for x, the equation becomes $8+20-64+36$, which is evidently $=0$; conſequently 2 and 2 are roots of it.

Otherwiſe, 2, the value of x given by the theorem, being written for it in the quadratic equation $3x^2+10x-32=0$, the reſult is $12+20-32=0$.

Or, dividing the given cubic by the quadratic $\overline{x-2}^2$, we have $x^2-4x+4)\,x^3+5x^2-32x+36\,(x+9$; therefore the three roots are 2, 2, and -9.

EXAMPLE II.

Given $x^3+\frac{10}{7}x^2-\frac{4000}{9261}=0$, an equation which has equal roots, to find them.

Here $q=0$, and the theorem gives $\frac{-36000\times40}{200\times9261}=\frac{-20}{21}$, which value being written for x the equation vaniſhes.

THEOREM

THEOREM II.

If the biquadratic equation $x^4 - px^3 + qx^2 - rx + s = 0$ *has two equal roots, make* $A = \frac{12r - 2pq}{3pp - 8q}$, $B = \frac{pr - 16s}{3pp - 8q}$, $C = \frac{4B - 2q}{4A + 3p}$, *and* $D = \frac{r}{4A + 3p}$, *and you will have* $x = \frac{D - B}{A - C}$.

A ſynthetical demonſtration of this theorem would be very long: the INVESTIGATION is as follows.

It has been demonſtrated by the writers on algebra, that, if a biquadratic equation, as $x^4 - px^3 + qx^2 - rx + s = 0$, has two equal roots, one of them may be had from the equation $4x^3 - 3px^2 + 2qx - r = 0$. Multiply this equation by x, and the original one by 4, and take the difference of the two, which will be $px^3 - 2qx^2 + 3rx - 4s = 0$. Again, if this equation be multiplied by 4, and the other cubic by p, and their difference taken, we ſhall have $\overline{3pp - 8q} \times x^2 + \overline{12r - 2pq} \times x + pr - 16s = 0$, or $x^2 + \frac{12r - 2pq}{3pp - 8q}x + \frac{pr - 16s}{3pp - 8q} = 0$, or $x^2 + Ax + B = 0$, putting A and B for the known quantities in the ſecond and third terms. Now multiply this equation by $4x$, and take the firſt cubic from it, and we ſhall have $\overline{4A + 3p} \times x^3 + \overline{4B - 2q} \times x + r = 0$, which being divided by $4A + 3p$, and C and D put equal to $\frac{4B - 2q}{4A + 3p}$ and $\frac{r}{4A + 3p}$ reſpectively, gives $x^3 + Cx + D = 0$; and this equation being taken from the other quadratic, there remains $\overline{A - C} \times x + B - D = 0$; conſequently $x = \frac{D - B}{A - C}$. *Q. E. I.*

COROLLARY 1. From the above inveſtigation it appears, that one of the equal roots may alſo be obtained from either of theſe two quadratic equations, of which the firſt ſeems moſt eligible,

as the co-efficients of it are leſs complex than thoſe of the other:

$$\overline{3pp-8q}\times x^3+\overline{12r-2pq}\times x+pr-16s=0,$$

and $\overline{4A+3p}\times x^2+\overline{4B-2q}\times x+r=0$. And theſe, when $p=0$, become $-8qx^3+12rx-16s=0$,

$$\text{and } -\frac{48r}{8q}x^2+\overline{\frac{64s}{8q}-2q}\times x+r=0,$$

$$\text{or } x^2-\frac{3r}{2q}x+\frac{2s}{q}=0,$$

$$\text{and } x^2+\frac{qq-4s}{3r}x-\frac{q}{6}=0.$$

COROL. 2. If both p and q vaniſh, then, from either of the quadratics we get $x=\frac{4s}{3r}$, perfectly agreeing with the cubic $px^3-2qx^2+3rx-4s=0$, which, when p and q vaniſh, becomes $3rx-4s=0$. And this equation is of uſe; becauſe, in this caſe, the theorem fails, one of the diviſors being $=0$.

COROL. 3. From the equation $4x^3-3px^2+2qx-r=0$, which, when p and q vaniſh, becomes $4x^3-r=0$, we alſo get $x=\sqrt[3]{\frac{r}{4}}$, another expreſſion of the ſame value of x.

COROL. 4. When $r=0$, $D=0$, and from the equation $x^3+Cx+D=0$, we have $x=-C$.

EXAMPLE I.

If the equation $x^4*-9x^3+4x+12=0$ has equal roots, it is propoſed to find them.

Here

Here $p=0$, $q=-9$, $r=-4$, and $s=12$; and

A becomes $=\frac{12\times-4}{-8\times-9}=\frac{-2}{3}$,

B . . $=\frac{-16\times-12}{-8\times-9}=\frac{-8}{3}$,

C . . $=\frac{4\times\frac{-8}{3}+18}{4\times\frac{-2}{3}}=\frac{-11}{4}$,

D . . $=\frac{-4}{\frac{-8}{3}}=\frac{3}{2}$,

and $\frac{D-B}{A-C}=\frac{\frac{3}{2}+\frac{8}{3}}{\frac{-2}{3}+\frac{11}{4}}=\frac{18+32}{-8+33}=\frac{50}{25}=2$,

which being written for x, the equation becomes $16-36+8+12=0$; therefore 2 is one of the roots.

The ſame value of x may be diſcovered from either of the quadratic equations mentioned in corollary 1. The proper values of the co-efficients being written in the firſt of them, it becomes $x^2-\frac{2}{3}x-\frac{8}{3}=0$, where one value of x is $\frac{1+\sqrt{25}}{3}=2$. The other quadratic becomes $x^2-\frac{11}{4}x+\frac{3}{2}=0$, one of whoſe roots is $\frac{11+\sqrt{25}}{8}=2$.

EXAMPLE II.

It being known that the equation $x^4-x^3-7x^2+13x-6=0$ has two equal roots, to find them.

Here $p=1$, $q=-7$, $r=-13$, and $s=-6$; and A$=\frac{-142}{59}$, B$=\frac{83}{59}$, C$=\frac{-1158}{391}$, D$=\frac{767}{391}$, D$-$B$=\frac{12800}{23069}$, A$-$C$=\frac{12800}{23069}$, and $\frac{D-B}{A-C}=\frac{12800}{12800}=1$, one of the roots ſought.

The

The ſame value of x may be found from either of the two general quadratic equations given in corollary 1. From the firſt of them we get one value of $x = \frac{71 - \sqrt{144}}{59} = 1$. And from the other, one value $= \frac{579 - \sqrt{35344}}{391}$, which is alſo $= 1$.

EXAMPLE III.

Given the equation $x^4 - \frac{1}{2}x + \frac{3}{16} = 0$, in which two values of x are equal to each other, to find them.

By corollary 2. we have $x = \frac{4 \times 3}{16} \div \frac{3 \times 1}{2} = \frac{8}{16} = \frac{1}{2}$. By corol. 3. x is $= \sqrt[3]{\frac{1}{8}} = \frac{1}{2}$.

THEOREM III.

If the ſurſolid equation $x^5 + px^4 + qx^3 + rx^2 + sx - t = 0$ *has two roots equal to each other, and you make* $A = \frac{15r - 3pq}{4pp - 10q}$, $B = \frac{2pr - 20s}{4pp - 10q}$, $C = \frac{25t - ps}{4pp - 10q}$, $D = \frac{5B - 3q}{5A + 4p}$, $E = \frac{5C + 2r}{5A + 4p}$, $F = \frac{s}{5A + 4p}$, $G = \frac{B - E}{A - D}$, $H = \frac{F + C}{A - D}$, $I = \frac{B - H}{A - G}$, *and* $K = \frac{C}{A - G}$, *then ſhall one of the equal values of* x *be* $= \frac{H - K}{I - G}$.

The inveſtigation of this theorem being altogether ſimilar to that of the laſt, it is unneceſſary to give it here.

The difference of equations being taken as in the inveſtigation of theorem II. it will appear, that one of the equal roots may alſo be had from any one of the following five equations, of which ſometimes one, ſometimes another, will be the moſt eligible.

1. $5x^4 - 4px^3 + 3qx^2 - 2rx + s = 0$.
2. $px^4 - 2qx^3 + 3rx^2 - 4sx + 5t = 0$.
3. $x^3 + Ax^2 + Bx + C = 0$.
4. $x^3 + Dx^2 + Ex - F = 0$.
5. $x^2 + Gx + H = 0$.

It is obvious, that, when p vaniſhes, the work will be conſiderably ſhortened; and when both p and q are wanting, though the above *formula* fails, yet the equal root may be eaſily obtained from the equation $px^4 - 2qx^3 + 3rx^2 - 4sx + 5t = 0$, which in that caſe becomes $3rx^2 - 4sx + 5t = 0$. Whenever s is wanting, F, in the ſecond cubic above, will be $= 0$, and conſequently x may be found from the quadratic equation $x^2 + Dx + E = 0$. But in any of theſe caſes the equal root may be found by diviſion. However, the operation probably will not, in general, be ſo ſhort as extracting the root of the quadratic; I will therefore haſten to give an example or two of the uſe of the theorem.

EXAMPLE I.

Given $x^5 + x^3 - x^2 + 0{\cdot}09433 = 0$, to find x, two values of it being equal to each other.

Here $p = 0$, $q = 1$, $r = 1$, $s = 0$, $t = -0{\cdot}09433$, and we get

$A = -1{\cdot}5$	$F = 0$
$B = 0$	$G = -0{\cdot}2231$
$C = +0{\cdot}2358$	$H = -0{\cdot}1241$
$D = +0{\cdot}4$	$I = -0{\cdot}0972$
$E = -0{\cdot}4238$	$K = -0{\cdot}185$

and $x = \frac{H-K}{I-G} = 0{\cdot}48$.

The proper values of the co-efficients being written in the five equations before mentioned, and ſome of them divided by the

the co-efficient of the higheſt power of x, we have theſe four equations, in each of which one value of x is one of the equal ones ſought:

$$x^3 * + 0{\cdot}6x - 0{\cdot}4 = 0.$$
$$x^3 - 1{\cdot}5x^2 * + 0{\cdot}2358 = 0.$$
$$x^2 + 0{\cdot}4x - 0{\cdot}4238 = 0.$$
$$x^2 - 0{\cdot}2231x - 0{\cdot}1241 = 0.$$

Now the moſt eligible equation is the quadratic $x^2 + 0{\cdot}4x - 0{\cdot}4238 = 0$, whoſe affirmative root is $\sqrt{0{\cdot}4638} - 0{\cdot}2 = 0{\cdot}4811$, agreeing with the value of x found above, but true to two places lower in the decimal.

EXAMPLE II.

To find the two equal values of x in the equation $64x^5 - 20x^2 + 3 = 0$.

The given equation being divided by 64, we have $x^5 - 0{\cdot}3125x^2 + 0{\cdot}046875 = 0$; and then, from the firſt of the five equations given above, we get $5x^4 - 0{\cdot}625x = 0$, and $x = \sqrt[3]{0{\cdot}125} = 0{\cdot}5$. But from the ſecond of the equations juſt mentioned, we have $0{\cdot}9375x^2 - 0{\cdot}234375 = 0$, or $x^2 = \frac{0{\cdot}234375}{0{\cdot}9375} = 0{\cdot}25$, and $x = \sqrt{0{\cdot}25} = 0{\cdot}5$.

From the foregoing few pages it is evident, that rules may be made for finding the equal roots of equations of more than five dimenſions by diviſion; but the operations by them will, in moſt caſes, be long and tedious. It is obvious, however, that ſuch equations may be depreſſed to any dimenſion the algebraiſt pleaſes.

It has indeed been ſuppoſed, that the number of equations that have equal roots is but ſmall, and, conſequently, that the chief

chief uſe of the rules for finding their roots is to get limits and approximations to the roots of equations in general. That uſe, it muſt be allowed, were it the only one, is ſufficient to pay for inveſtigating them. But if the equations that have equal roots ſhould hereafter be found not ſo few as has been generally received, then the uſe of the above theorems will become more extenſive.

I beg leave to add, that this ſhort eſſay is but a ſmall part of a work, in which, if I ſhould ever have leiſure to put a finiſhing hand to it, ſomething more on this ſubject may very probably appear. In the mean while, I hope, this little piece will be candidly received by thoſe who have more leiſure and better abilities for ſtudies of this kind.

Conſtantine,
February 9, 1782.

XXVII. *Some farther Conſiderations on the Influence of the Vegetable Kingdom on the Animal Creation.* By John Ingen-houſz, *Counſellor to the Court, and Body Phyſician to the Emperor, F. R. S. &c.*

Read June 13, 1782.

UPON being informed, a few months ago, as well by private letters as from the Critical Review, that my doctrine was quite over-turned by the fifth volume of Dr. PRIESTLEY, and by an experiment quoted in the book of Mr. CAVALLO on Air; I invited ſome of my friends here to aſſiſt at ſome deciſive experiments, of which I will here give an exact account. I told them the whole reſult which was to be expected from them, if my ſyſtem was founded on nature, explaining to them before-hand the theory of theſe reſults, and promiſing, at the ſame time, that, if the reſult ſhould fail, I ſhould myſelf be the firſt to diſcredit my own ſyſtem. I had the ſatisfaction to convince them that the reſult did fully anſwer my prediction and expectation. Theſe experiments are the following, all made in a hot-houſe of the Botanical Garden in the winter of 1782.

I expoſed to the ſun-ſhine ſix globular glaſs veſſels, each containing about 160 cubic inches of ſpace, all filled with pump-water, which was boiled during more than two hours, and poured quite hot into the glaſs veſſels, on purpoſe to prevent any acceſs of air to the water.

Exp. I. In two of these vessels I put as much of the *conferva rivularis* (a water plant, classed by LINNÆUS amongst the *cryptogamia*) as was sufficient to take up the space of about an inch square.

Exp. II. In the two next vessels I suspended by threads tied to bits of cork, some pieces of woollen and silk cloth of different colours, as white, scarlet, green, and brown, having previously wetted them in some boiled water, on purpose to free them from all air.

Exp. III. In the two remaining vessels I placed nothing at all.

Exp. IV. In another vessel of the same form and size I put some of the *conferva rivularis*, and filled it with pump water.

All these globular vessels were inverted, with their orifices immersed in vessels filled with quicksilver, for the purpose of preventing effectually any communication between the contents of the vessels and the atmosphere.

Result of experiment I. The first two days neither of the vessels contained any air, and even the small quantity of air, which here and there adhered in the form of a bubble to the fibres of the vegetable when it was shut up in the vessel, had entirely disappeared. The third day, in the morning, some air bubbles began to rise from every part of the *conferva* in both glasses; and in the afternoon of the same day, a great quantity of air bubbles rose continually from it. I took at that time the vegetable out of one of these vessels. I plunged a wax taper, just extinguished, into the orifice of this vessel, on purpose to see whether the air, already extricated from the *conferva*, was dephlogisticated or not. The wax taper took flame immediately with an uncommon splendour. After this I poured the half of the water from the globular vessel into a common bottle, and corked it. I inverted this bottle afterwards

in an earthen veſſel filled with boiled water. I placed this apparatus near the fire till the water in the bottle began to boil; after which I cooled the whole, and found a good quantity of air collected in the bottle, which air proved to be dephlogiſticated. When I drew the vegetable out of the glaſs veſſel I obſerved the water to ſparkle almoſt like Seltzer water, or like water impregnated by art with fixed air. The vegetable which was ſtill kept in the ſecond bottle of exp. I. continued to yield air in the ſun-ſhine, till it ceaſed to throw up any more air, towards the ſeventh or eighth day of its being ſhut up in the veſſels. When, after this time, this globular veſſel was ſhook, the water became full of ſmall air bubbles, which for the moſt part roſe to the inverted bottom of the veſſel, great part of them ſettling upon the vegetable, which appeared all covered with them. This ſparkling air, which became viſible by ſhaking the glaſs, could not but be air originally produced by the *conferva*, and ſo looſely joined with the water, that it diſengaged itſelf in a great meaſure from it by the motion of the veſſel. After the tenth day the vegetable began to appear withered, grew yellow, and began to die. I found about eight cubic inches of dephlogiſticated air collected in the veſſel. This proved to be of a very eminent quality, its goodneſs being of 352°; that is to ſay, that from a mixture of one meaſure of this air, and as many meaſures of nitrous air as were neceſſary to complete the full ſaturation, there were deſtroyed three meaſures and $\frac{52}{100}$ of a meaſure, the teſt being made with Abbé FONTANA's Eudiometer, employed in the manner deſcribed in my book upon Vegetables, p. 278. *et ſeq.* The quality of this air was ſuperior to that of any air I ever got from this plant in freſh pump water, its goodneſs proving, in general, to be from 260 to 330°, in the hot-houſe; this was during the winter, for I never had

been

been able to obtain such fine air from this vegetable in the summer*; the reason of which I will explain elsewhere.

Theory of exp. I. Boiled water, having lost its air, is very much disposed to imbibe it from all bodies which contain this fluid; and therefore, during the first day or two of its exposure to the sun-shine with the vegetable, this water absorbed all the air which the plant emitted; and even that which had remained entangled between the fibres of the vegetable when it was immersed in this water. The water being at last saturated with this air could take up no more; and therefore, whatever air, after this saturation, came forth from the vegetable rose to the top of the vessel. The quantity of this dephlogisticated air was smaller than that which an equal bulk of the same plant commonly yields in fresh pump water, because a great deal of air was at first taken in or absorbed by the boiled water; which absorption does not happen, or at least is not so great,

* By continuing to make experiments, during the whole winter, in the hot-houses of the botanical garden, I found that the *conferva rivularis* yielded dephlogisticated air of a much superior quality to that I had ever been able to get from it in the summer, in the open air; whereas those plants, such as the *agave Americana*, *cactus triangularis*, &c. which yielded in the summer the best air, did scarce yield any in the winter (and that of a quality scarce better than common air) though placed next to the *conferva*. The quantity of dephlogisticated air I got in the winter from the *conferva* was so great, that as much of this vegetable as occupies about the space of one cubic inch commonly yielded from 12 to 16 cubic inches of this air in the space of three or four days, when the sun did shine, the quantity of pump-water being about 160 cubic inches. The green matter which Dr. PRIESTLEY mentions as spontaneously produced from pump-water, gave in the winter also a tolerable quantity of dephlogisticated air, of a good quality, though not so fine, or in such large quantities, as it is used to give in the summer. It seems to be a general rule, that the greater the quantity of air obtained from vegetables in the sun is, the better is its degree of goodness.

when

when pump water is uſed, as this water is always nearly ſaturated with air. The air, thus obtained in boiled water, was of a finer quality than that commonly obtained by the ſame means in pump water; becauſe this air, being entirely free of air diſengaged from freſh water, muſt of courſe be ſo much the purer dephlogiſticated air. The water of the firſt veſſel, and which was taken out of it as ſoon as air bubbles began to riſe from the plant, ſparkled like Seltzer water, and yielded by heat dephlogiſticated air, becauſe it was then already ſaturated with dephlogiſticated air iſſuing from the vegetable. The water of this veſſel being ſhook, after the vegetable ceaſed to throw up any more air in a viſible way, ſtill continued to ſparkle; becauſe, though the vegetable by loſing gradually its vigour, was at laſt no more able to throw up air in viſible bubbles, yet it had ſtill enough of its vital power left to keep the water ſaturated with dephlogiſticated air, ſo as to ſparkle when ſhook. This vegetable, continually robbing the water of its natural air, found at laſt nothing more in it to ſupport its life, and therefore at laſt languiſhed and periſhed; which it did ſo much the ſooner, from the contact of the dephlogiſticated air, with which the water was impregnated, and to which the vegetable had been all that time expoſed, having hurt its conſtitution (it is well known, that plants die in dephlogiſticated air) and thus haſtened its death. The ſparkling quality of the water did not ceaſe entirely till the vegetable was quite deprived of its life. The water began to ſparkle every day very briſkly, by being ſhook after the apparatus had been expoſed an hour or two to the ſun-ſhine, during the time the vegetable was in its full vigour, and ceaſed to do ſo ſome time after ſun-ſet, or after the apparatus had been withdrawn from the ſun's light; becauſe this vegetable, like all others, elaborates no dephlogiſticated air but by the

influence of the ſun; and becauſe this air, being but looſely united with the water, diſengaged itſelf again from it, and roſe to the inverted bottom of the veſſel. The water being thus deprived of the dephlogiſticated air, ceaſed to ſparkle any more, till it became again ſaturated with it, after the apparatus had been expoſed again, during ſome time, to the influence of the light.

Reſult of exp. II. No air at all was produced in the veſſel containing the pieces of cloth, during three weeks expoſure to the ſun-ſhine.

Theory of exp. II. Boiled water, having loſt its air, could yield none, at leaſt till after a long time, when ſome degree of corruption took place in the animal ſubſtance, *viz.* the pieces of cloth.

Reſult of exp. III. Not an atom of air has appeared in this veſſel, though it ſtood about two months upon the ſame place.

Theory of exp. III. Boiled water having no air, the ſun could extricate none from it.

Reſult of exp. IV. the *conferva* began to yield air bubbles the very ſame day, a little while after its expoſure to the ſun. The next day it threw up an immenſe quantity of them. The fifth day it begun to throw up leſs, and ceaſed entirely about the ſeventh day, when the quantity of about fourteen cubic inches of dephlogiſticated air, of an excellent quality, though leſs fine than that obtained in exp. I. was collected. The water ſparkled, as does Seltzer water, by the veſſel being ſhook. This water being expoſed to the fire, in an inverted veſſel, yielded a good quantity of air, which was ſo far dephlogiſticated as to be able to kindle a wax taper juſt extinguiſhed. After the tenth day the vegetable began to die.

Theory

Theory of exp. IV. The vegetable threw up very ſoon air bubbles, becauſe this water, being in its natural ſtate, and thus ſaturated with air, could not abſorb much of the air iſſuing from the vegetable, which air muſt, of courſe, ſoon riſe up in viſible bubbles. A great deal more air was collected than in exp. I. becauſe leſs of the air iſſuing from the plant was abſorbed by this water than in exp. I. The air obtained was not ſo good as that obtained in exp. I.; becauſe the air in this experiment was ſomewhat infected by the air iſſuing from the water, which was but common air. The water ſparkled when the veſſel was ſhook, becauſe this water, though it had probably loſt ſome of its own air, yet had aſſumed a great deal of air from the plant *, which air diſengages itſelf from the water very eaſily, juſt as fixed air does; the more ſo when the water is moved.

* It ſeems, that dephlogiſticated air has by no means ſuch a ſtrong attraction to water, in other words, has not ſo much affinity with it, as common air has: it appears to be but looſely united with water, and quits it very eaſily by the water being ſhaken, or even though the water be kept quiet. May not this quality be looked upon as a providential one? for thus the dephlogiſticated air, produced by water plants, is continually emitted by the water, and diffuſed through the common ſtock; and for this reaſon the water, when ſhook during the day-time, always ſparkled as champaigne; becauſe at that time the water was always kept ſaturated by the dephlogiſticated air, iſſuing continually out of the vegetable. But the water ceaſed to ſparkle, after having been withdrawn from the ſun-ſhine during ſome hours; becauſe the dephlogiſticated air, with which the water was ſaturated during the day-time (ſuppoſing the apparatus to be kept expoſed to the ſun) being but looſely united with the water, diſengaged itſelf gradually from it, and roſe to the top of the veſſel. The ſparkling quality of this water returned after the veſſel had been expoſed afreſh to the ſun; becauſe the vegetable, reſuming in that expoſition its daily operation, communicated a freſh ſupply of dephlogiſticated air to the water. This ſparkling quality returned every day, as long as there remained any life in the vegetable; after which the water ſparkled no more (though ſhook ever ſo much) either by day or in the night.

This water yielded, by heat, true dephlogiſticated air; whereas the ſame water, when it has not been expoſed to the action of a vegetable, yields by heat nothing but common air. The reaſon of it is, that the air elaborated by the plant, with which this water was ſaturated, was real dephlogiſticated air. The vegetable at laſt languiſhed and began to die, becauſe the water was impregnated with dephlogiſticated air, which being an excrement of the plant is hurtful to its conſtitution. Beſides, this water had at laſt loſt the moſt part of, or perhaps all, its own ſtock of common air; and with this all the nutritive nouriſhing and phlogiſtic particles, which were taken in by the plant, and was therefore become leſs fit to keep up vegetable life.

All theſe experiments were repeated frequently, and always with the ſame general reſults *.

I think the abovementioned facts will be looked upon as quite ſufficient to put my doctrine out of all farther queſtion. I have many more facts, perhaps equally demonſtrative with thoſe juſt deſcribed; but, as this paper is already too long, I will keep them for ſome other opportunity. However, I cannot forbear making ſome farther remarks by which the point in queſtion may be ſtill farther illuſtrated.

If it was the water, and not the vegetable, which yielded the dephlogiſticated air; and thus, if the reaſon why water plants and the green matter ceaſe at laſt to throw up more air (if the

* That air is thrown out of living vegetables expoſed to the ſun-ſhine, was already obſerved by the rev. Dr. HALES, as may be ſeen in vol. I. of his Statical Eſſays, p. 110. The apparatus which he uſed for this experiment is repreſented by fig. XVII. plate VII. But this ineſtimable philoſopher, not even ſuſpecting that this air was of a peculiar nature, did not collect it. BOYLE obtained much air from vegetables *in vacuo*.

water is not renewed) is, that the water being at last exhausted of its air can yield no more, it would necessarily follow, that the water, thus supposed to be deprived of air, should be similar to distilled or boiled water; but it is quite the reverse. This very water, instead of being exhausted of air, gives evident signs of being over saturated with it. It sparkles almost as champaign does, when shook by day-time in the sun; and it will, when it is at that time separated from the vegetable, yield by itself in the sun, but more so by the fire, a good quantity of dephlogisticated air. The reason why the green matter ceases at last to yield more air is therefore by no means because the water is exhausted of air; but, on the contrary, because it is too much saturated with it, and that with an air hurtful to vegetable life, and because this water has at last lost its own natural air, and together with this air the nourishing and phlogistic particles which are necessary to keep up the full vigour of plants.

If it was the water, and not the vegetable, which furnished the dephlogisticated air, why should the air bubbles not settle indifferently on either surface of the leaves? In this supposition, how could that admirable regularity be accounted for, by which all the leaves of a vine and a lime-tree are first covered with air bubbles on the under side, and all the leaves of *lauro-cerasus*, at the upper surface, whichever surface of the leaves is exposed to the rays of the sun? Why should the air always settle on most leaves in the form of bubbles, but never so upon the leaves of the *tropæolum majus*, at least during the first hours, but always in the form of bags, adhering to the upper edge of the leaves, and detaching themselves when they are grown to a certain size,

ſize, which never happens with leaves of a vine or lime-tree *; whereas the green ſtalks of this plant (the *tropæolum majus*) are at the ſame time all covered with ſeparate air bubbles? More of theſe remarkable appearances in different plants are to be ſeen in the third ſection of my book on Vegetables.

If the dephlogiſticated air, obtained by means of vegetables in water, was air depoſited from the water, and purified of its phlogiſton, by remaining, during a certain time, in contact with the vegetable, it would follow, that this air would be ſo much the purer the longer it remains in contact with the vegetable; but it is quite the reverſe. The air obtained from the leaves of a vine, to which the air bubbles ſtick a long while before they detach themſelves and riſe up, is never by far ſo much dephlogiſticated as is the air obtained from ſome American plants, out of which the air ruſhes almoſt all in continual ſtreams, as ſo many ſprings, not remaining a ſingle moment upon the leaves. The difference of both airs is ſo great, that I never got dephlogiſticated air from leaves of a vine, lime-tree, and ſuch like, whoſe goodneſs ſurpaſſed 260° (it is commonly a little above 200°); whereas I got commonly, from the above mentioned American plants, an air whoſe goodneſs was of above 300°, ſometimes even of above 350°, in a very fair day, this air being put to the teſt, according to the manner which I have deſcribed before †.

If

* I have explained the reaſon of this ſingularity in that plant, in my book, where it is mentioned by the name of *naſturtium indicum*.

† The *ſempervivum tectorum*, which grows almoſt every where upon the roofs of houſes, gives alſo a very great quantity of dephlogiſticated air of an eminent quality. It ſhould ſeem, that all fleſhy plants particularly excel in the quality of yielding fine dephlogiſticated air, and a great quantity of it. The *agave Americana*

If it was the water, and not the plant, which yielded the dephlogisticated air, the quantity of air obtained would bear, in general, a proportion to the quantity of the water employed; but this is by no means the case. The quantity of air bears a proportion to the bulk of the vegetable much more than to the quantity of the water. This is very easily to be observed with some of the above mentioned American plants. It ought to be always observed, that if too many leaves are crowded together, they shade one another too much; and therefore, in this case, the quantity of air obtained will be proportionably less, and its quality worse.

Again: if the dephlogisticated air, obtained from plants in water, was air disengaged from the water, it would follow, that a plant shut up in a transparent glass vessel without any water would yield no air at all, nor increase the quantity of air shut up with the vegetable. The following experiment, I think, will be sufficient to convince any one that this is far from being the case. I placed in a glass tube, hermetically sealed at one end, a piece of an American plant, called *cereus*; the extremity of this piece, where it was cut from the plant, was tightly squeezed in a small glass vessel, in which only as much water was kept as seemed to be required to keep the *cereus* in full vigour. I smeared the vegetable, and the orifice of the glass vessel all around with soft wax, so that all communication between the air within the tube and the water within the small vessel was cut off. I placed this tube inverted in a vessel filled with quicksilver, keeping a column of some inches of

ricana gives such a prodigious quantity of dephlogisticated air, that, in a fair day, I often got from one single leaf above 150 cubic inches of this air, of the finest quality. The *cactus triangularis*, *cereus*, *sempervivum arborum*, and many others give no less air.

the metallic fluid in the tube, to allow the air within the tube to expand by the heat of the ſun without eſcaping. After this apparatus had ſtood during a few hours, in a bright ſun-ſhine, I cooled the whole to the ſame degree it poſſeſſed when it was expoſed to the ſun. This was done by plunging the whole in a tub filled with water, whoſe degree of heat was reſtored to the ſame degree it had before. I found the quantity of air within the tube remarkably increaſed, and ſo far dephlogiſticated that a flame burned in it with an increaſed brightneſs, and that one meaſure of it joined to one meaſure of nitrous air occupied 0.64; whereas the ſame air, before it was ſhut up with the *cereus* was of ſuch a degree of goodneſs, that one meaſure of it with one of nitrous occupied 1.06. Though this experiment may fully ſhew that plants throw out air in the ſun-ſhine, yet if another plant, which does not by nature yield ſo much air as the *cereus* commonly does, is ſhut up in a ſimilar tube, and expoſed to the ſun, the ſame effect will not always be obtained. The reaſon of it is, that plants abſorb a good quantity of common air as their nouriſhment at the ſame time that they throw out dephlogiſticated air. This fact, therefore, if conſidered by itſelf, will not be looked upon as equally demonſtrative with the above mentioned ones. The reſult of this experiment may depend on the more or leſs vigour of the vegetable employed, on the more or leſs brightneſs of the ſun's light, on the more or leſs heat the vegetable receives, &c. A *cereus* being a plant of hot climates may bear more heat than an European plant. All theſe circumſtances, as well as many others, may make the iſſue of this experiment ſometimes ambiguous; but the fact, as I have related it, joined to the above mentioned analogous experiments

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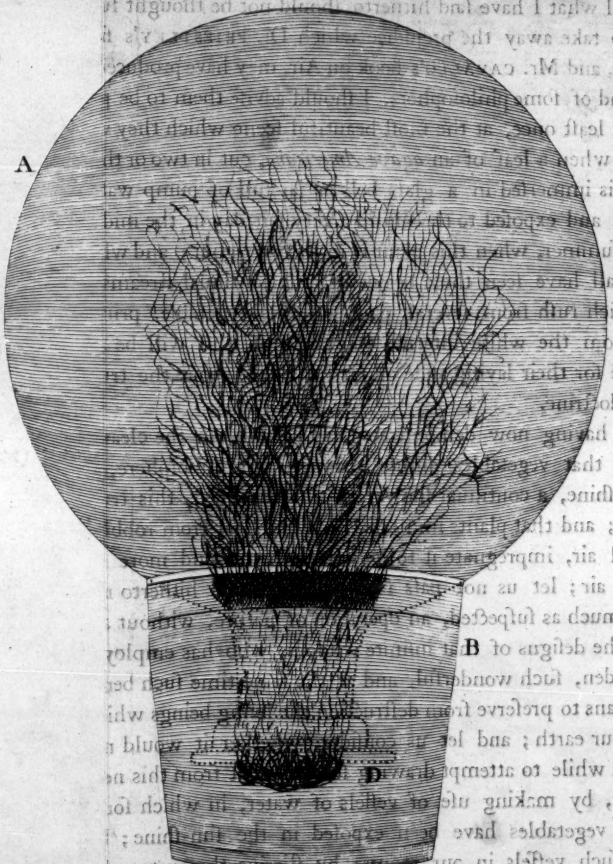
of Dr. HALES and Mr. BOYLE, will add ſtrength to my aſſertion, *viz.* that vegetables really throw out air in the ſun-ſhine.

If all what I have ſaid hitherto ſhould not be thought ſufficient to take away the prejudice which Dr. PRIESTLEY's fifth volume, and Mr. CAVALLO's book on Air, may have produced in the mind of ſome philoſophers, I ſhould adviſe them to be preſent, at leaſt once, at the moſt beautiful ſcene which they will behold, when a leaf of an *agave Americana*, cut in two or three pieces, is immerſed in a glaſs bell or jar full of pump-water, inverted and expoſed to the ſun in a very fair day in the middle of the ſummer, when this plant is in its full vigour; and when they ſhall have ſeen thoſe beautiful and continual ſtreams of air, which ruſh from ſeveral parts of this vegetable, principally from the white internal ſubſtance of it, I will be anſwerable for their laying aſide all farther doubt about the truth of my doctrine.

After having now demonſtrated, as I think, in the cleareſt manner, that vegetables diffuſe through our atmoſphere, in the ſun-ſhine, a continual ſhower of this beneficial, this truly vital air; and that plants immerſed in water, far from robbing it of all air, impregnate it fully with a better and more ſalubrious air; let us not paſs ſo wonderful, and hitherto not even ſo much as ſuſpected, an operation of nature, without admiring the deſigns of that infinite wiſdom, who has employed ſuch hidden, ſuch wonderful, and at the ſame time ſuch beneficial means to preſerve from deſtruction the living beings which inhabit our earth; and let us conſider, whether it would not be worth while to attempt drawing ſome benefit from this new diſcovery, by making uſe of veſſels of water, in which ſome leaves of vegetables have been expoſed in the ſun-ſhine; by placing ſuch veſſels in our rooms; by ſtirring the water; by



A. *a globular glass vessel containing about 160 cubic inches of water.*
B. *glass vessel filled with Mercury in which the orifice of the spherical vessel is plunged.*
C. *a vegetable called Conferva rivularis.*
D. *a piece of wood to which the Conferva rivularis is attached to keep it in its place.*

ſprinkling with it our floors inſtead of uſing for this purpoſe common water; by placing within our houſes, inſtead of flower-pots, diſhes containing ſome *conferva rivularis*, a plant to be met with almoſt every where, ſhooting forth with the utmoſt luxuriancy in all water baſons, in all tubs and veſſels in which water is kept. Is it poſſible, after all this, not to believe, that the Creator has multiplied this vegetable with a ſimilar view to our benefit? This benefit we may now, with ſome confidence, apply to our preſervation, by honouring this vegetable with a place in thoſe of our own rooms which are expoſed to the ſun, and keeping it alive as long as we pleaſe; which may be done by only pouring every day freſh water upon it, and ſqueezing gently now and then out of it the dephlogiſticated air with which the whole maſs ſwells up almoſt as ſoon as the ſun caſts its rays upon it. The water itſelf, in which it has been immerſed, will now, perhaps, be looked upon as too precious to be thrown away as uſeleſs and deprived of that very principal of animal life, of which I have demonſtrated it to be highly pregnant.

XXVIII. *A Microſcopic Deſcription of the Eyes of the* Monoculus Polyphemus LINNÆI. *By Mr.* William André, *Surgeon; communicated by Sir* Joſeph Banks, *Bart. P. R. S.*

Read May 30, 1782.

THE wonderful ſtructure of the eyes of inſects in general, moſt commonly illuſtrated by that of the *Libellula*, or *Dragon-fly*, cannot fail of ſtriking with aſtoniſhment the naturaliſt who inveſtigates the works of the great Creator in his moſt minute productions. According to LEWENHOEK, HOOK, and others, the *corneæ* of moſt inſects are made up of an infinite number of ſmall, tranſparent, horny lenſes, each reſembling, in ſome degree, a ſmall magnifying glaſs. This ſtructure prevails in the *corneæ* of inſects in general; but the *Monoculus Polyphemus*, or *King Crab*, is, among others, an exception to this rule.

The *Monoculus Polyphemus*, or *King Crab*, is a cruſtaceous animal found in all the ſeas ſurrounding the continent of America and the Weſt India iſlands, and which frequently grows to a very large ſize *. I ſhall deſcribe ſo much of the *Monoculus* only as is neceſſary to point out the ſituation of the eyes, which have been looked upon as two in number only †, though in reality they are four. The largeſt piece of the cruſtaceous covering of this animal, when ſeparated from the reſt of the ſhell, has very much the ſhape of a barber's baſon, or the fore-

* Bossu's Travels, vol. I. p. 368.

† LINNÆI Syſtema Naturæ, tom. I. p. 1057.

part

part of a woman's bonnet. The eyes are a part of the ſhell, or, as LINNÆUS expreſſes it, they are *teſtæ innati**. They may be diſtinguiſhed by the terms large and ſmall, or lateral and anterior. If the ſhell were divided fairly in half, the large eyes would be nearly in the center of each piece, and the ſmall ones on the divided edge near the fore-part of the ſhell. The large eyes are at a great diſtance from each other; but the ſmall ones are cloſe together. It will appear hereafter, that the large eyes are made up of a great number of ſmall, tranſparent, amber-like cones, and that the ſmall ones are compoſed of one ſuch cone only; ſo that they may be divided into eyes with many cones, and eyes with a ſingle cone †. The large eyes, or thoſe with many cones, appear as two tranſparent ſpots about the ſize and nearly of the ſhape of a kidney bean, the concave edges looking towards each other, and the convex towards the edge of the ſhell. If they be examined attentively, we may diſcern on their ſurface a number of ſmall depreſſions, which point out the center of each cone. The ſmall eyes, or thoſe with a ſingle cone, look like two ſmall tranſparent ſpots, not larger than a pin's head; theſe, from their minuteneſs, are eaſily overlooked, ſee fig. 1. where A.A. ſhew the large eyes, and B.B. the ſmall ones.

The appearances which I have deſcribed may be ſeen on the external ſurface of the ſhell with the naked eye; but in order to proceed to a further inveſtigation of the ſubject, the *corneæ*

* This being the caſe, the eyes can enjoy no motion; in which particular, as well as in ſome others, the *Monoculus Polyphemus* differs from the genus of crabs, whoſe eyes are placed on petioles, or ſtalks, and are moveable.

† The Greek words πολυς κωνος, and μονος κωνος, would expreſs the ſenſe in a more conciſe manner. *Oculi polyconici et oculi monoconici.*

muſt be removed from the ſhell, and applied to a ſingle microſcope with a very ſtrong light.

The internal ſurface of the large eyes, examined with the microſcope, is found to be thick ſet with a great number of ſmall, tranſparent cones, of an amber colour *, the baſes of which ſtand downward, and their points upwards next the eye of the obſerver †. The cones in general have an oblique direction, except ſome in the middle of the *corneæ*, about thirty in number, the direction of which is perpendicular. The center of every cone being the moſt tranſparent part, and that through which the light paſſes; on that account the perpendicular or central cones always appear beautifully illuminated at their points. In a word, they are all ſo diſpoſed as that a certain number of them receive the light from whatever point it may iſſue, and tranſmit it to the immediate organ of ſight, which we may reaſonably ſuppoſe is placed underneath them; but this laſt circumſtance can only be determined in a recent ſubject, which I have never been ſo lucky as to ſee. The cones are not all of the ſame length; thoſe on the edges of the *cornea* are the longeſt, from whence they gradually diminiſh as they approach the center, where they are not above half the length of thoſe on the edges, ſee fig. 2.

As theſe cones ſo eaſily tranſmit the light through their ſubſtance, when I firſt examined them I thought they were tubes; but I have ſince viewed them broken in different directions, and am convinced they are ſolid tranſparent bodies. If they be viewed with a deep magnifier, every cone appears divided tranſverſely by two or three internal *ſepta* or partitions.

* I have made ſome attempts to aſcertain their number, and think they amount to about 1000.

† This muſt be reverſed if the eye be conſidered in its natural poſition.

This appearance is owing to the cones themſelves being made up of ſeveral cones, one within another, the *ſepta* or partitions being nothing more than the *apices* or points of the external cones; but this will be further explained by conſidering that the *cornea* of the *Monoculus* may be divided into layers, the number of which, however, I cannot aſcertain; but I once met with a *cornea* in which the internal layer and its cones was ſeparated from the external *lamina* and their cones. A portion of the interna. layer is ſhewn fig. 4.; and the cones, very much magnified, with their *ſepta* or partitions, are exhibited fig. 5.

It is very well known, that all cruſtaceous animals depoſit their ſhells once a year, and are left with a ſoft, tender covering, which, after ſome time, acquires the hardneſs of the former ſhell. As the *cornea* in theſe animals is a part of the ſhell, it is reaſonable to ſuppoſe, that the internal layer is left with the ſoft covering, containing the rudiments of the future *cornea*; and this is the more probable, from what I have before obſerved, that I have met with an eye where the internal layer was ſeparated from the more external ones, ſee fig. 4.

The ſtructure of the ſmall eyes being leſs elaborate than that of the large ones, their internal appearance, when placed in the microſcope, will be deſcribed in a few words. They conſiſt of an oval, tranſparent, horny plate, of an amber colour, in the center of which ſtands a ſingle cone, through which and the oval plate the light paſſes, ſee fig. 3 *.

Having thus deſcribed, as conciſely as poſſible, the ſingular mechaniſm of the *corneæ* of the *Monoculus*, I ſhall add a few words concerning their uſe. The lenticular ſtructure of the *corneæ* of inſects in general certainly aſſiſts in condenſing or

* The ſmall eyes are analogous to thoſe ſmall eyes of other inſects which entomologiſts have called *ſtemmata*.

ſtrengthening the light in its paſſage to the immediate organ of ſight. It is probable, that the cones in the *Monoculus* have the ſame effect. Whether they anſwer that purpoſe, in a more or leſs perfect manner than the lenſes in the generality of inſects, is what I cannot take upon me to determine.

EXPLANATION OF THE PLATE.

Fig. 1. The *Monoculus Polyphemus.*

AA. The large eyes.

BB. The ſmall ones.

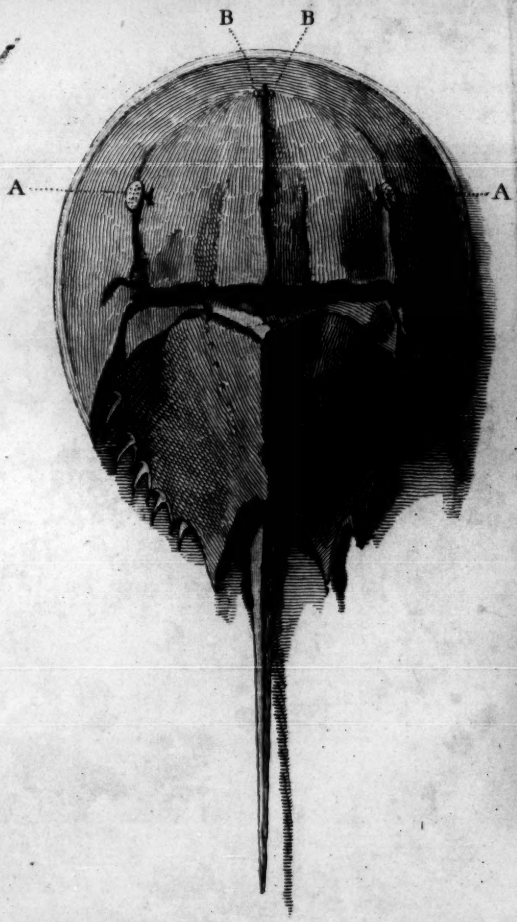
Fig. 2. One of the large eyes magnified.

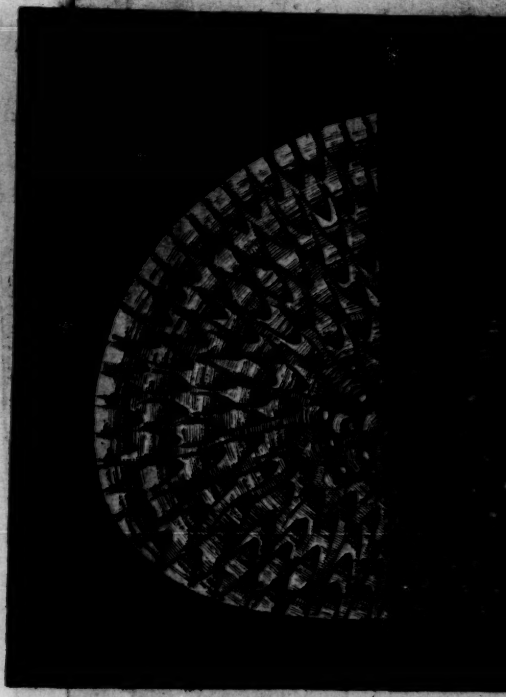
Fig. 3. One of the ſmall eyes magnified.

Fig. 4. A portion of the internal layer magnified.

Fig. 5. The cones magnified with their *ſepta* or partitions.

Fig.1.





MONOCULUS P

Fig. 2.

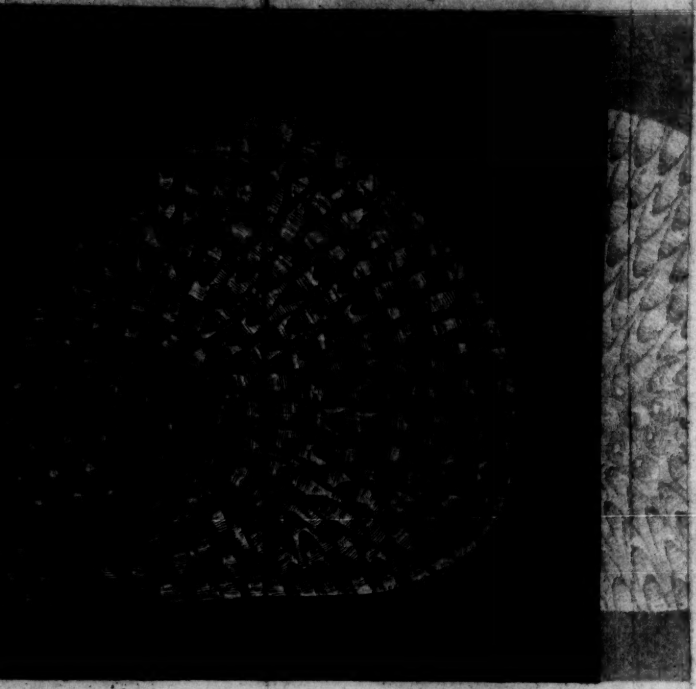


Fig. 5.

Fig. 3

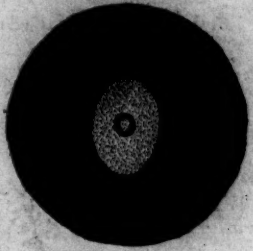
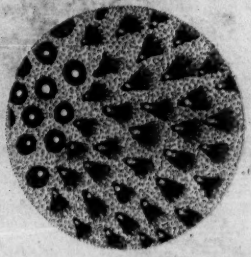


Fig. 4.



S Polyphemus *LINNÆI.*

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77.	12.	*for* mercury removed without causing this to rise *read* removed without causing the mercury to rise
81.	9.	*for* ſmaller towards the middle *read* larger towards the middle
391.	—	the laſt line, *for* $-$ *read* $+$
403.	12.	*for* -54 *read* -46
408.	3.	from the bottom, *for* $\frac{rn+m}{n}$ *read* $\frac{n}{rn+m}$